CONTENTS

Lecture

1

1.1 Introduction:

In everyday life, it is seen that a number of people arrive at a cinema ticket window. IF the people, arrive "too frequently" they will have to wait for getting their tickets or sometimes do without it. Under such circumstances, the only alternative S to form a queue, called the waiting line, in order to maintain a proper discipline. Occasionally, it also happens that the person issuing tickets will have to wait, (I. e. Remains idle), until additional people arrive. Here the arriving people are called the customers and the person issuing the tickets is called a server

Another example is represented by letters arriving at a typist's desk. Again, the letters represent the customers and the typist represents the sever; A third example is illustrated by a machine breakdown situation. A broken machine represents a customer calling for the service of a repairman. These examples show that the term customer may be interpreted in various numbers of ways. It is also noticed that a service may be performed either by moving the server to the customer or the customer to the server

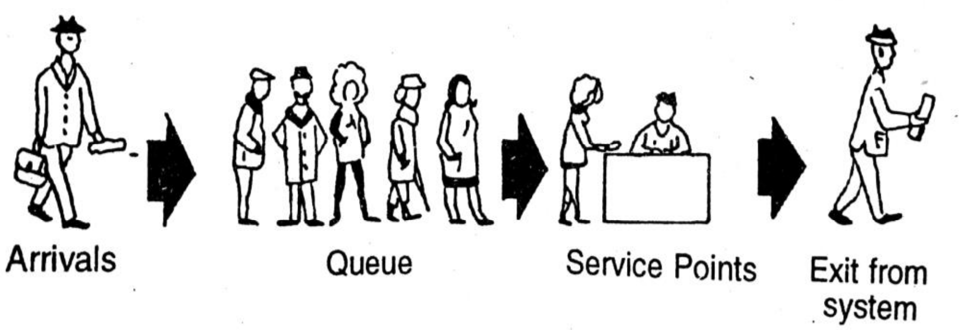

Fig.1

Thus, it is conclude that waiting lines are not only the lines of human being but also the aero planes seeking to land at busy airport, ships to be unloaded, machine parts to be assembled, cars waiting for traffic lights to turn green, customers waiting for attention in a shop or supermarket, calls arriving at a telephone switch-board, jobs waiting for processing by a computer, or anything else that require work done on and for it are also the example of costly and critical delay situation. Further, it is also observe that arriving units may form one line and be serviced through only one station (as in a doctor's clinic), may form one line and be served through several stations (as in a barber shop), may form several lines and be served through as many station(e.g. At check supermarket).

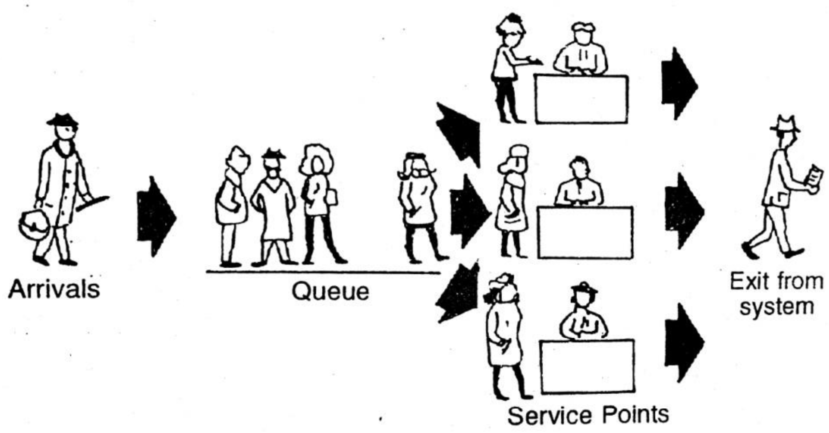

Fig.2

Servers may be in parallel or in series. When in parallel, the arriving customers may form a single queue as shown in Fig. 3 or individual queues in front of each server as is common in big post-offices. Service times may be constant or variable and customers may be served singly or in batches (like passengers boarding a bus).

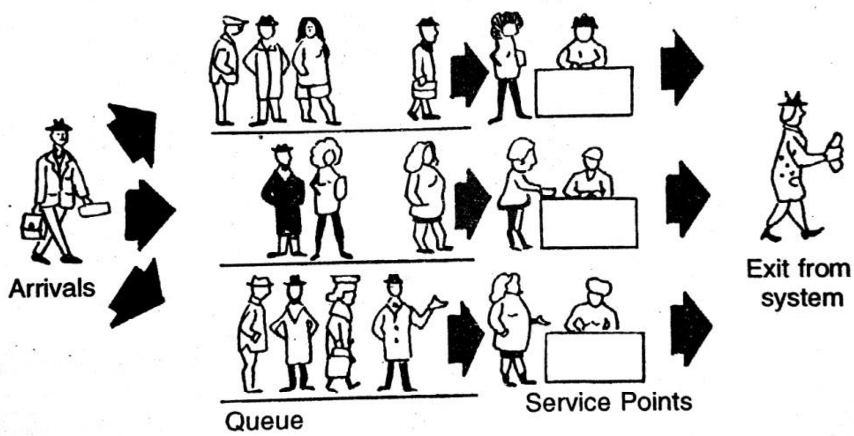

Fig.3

Fig. 4 illustrates how a machine shop may be thought of as a system of queues forming in front of a number of service centers, the arrows between the centers indicating possible routes for jobs processed in the shop. Arrivals at a service centre are either new jobs coming into the system or jobs, partially processed, from some other service centre. Departures from a service centre may become the arrivals at another service centre or may leave the system entirely, when on these items processing on these items is complete.

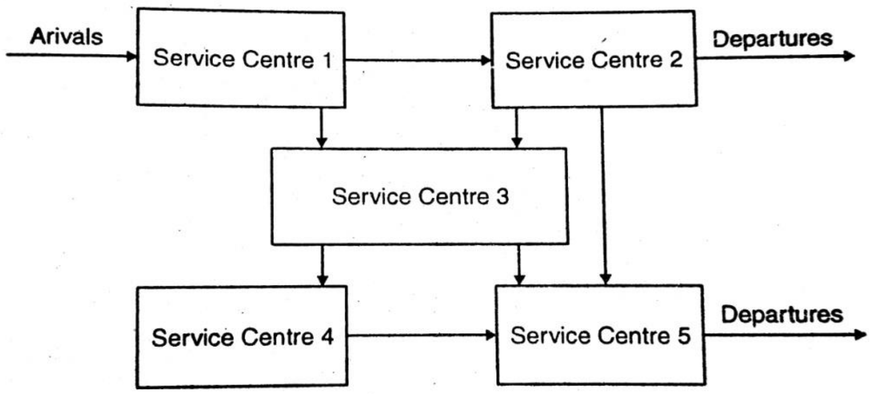

Fig.4

Queuing theory is concerned with the statistical description of the behavior of queues with finding, e.g., the probability distribution of the number in the queue from which the mean and variance of queue length and the probability distribution of waiting time for a customer, or the distribution of a server's busy periods can be found. In operational research problems involving queues, investigators must measure the existing system to make an objective assessment of its characteristics and must determine how changes may be made to the system, what effects of various kinds of changes in the system's characteristics would be, and whether, in the light of the costs incurred in the systems, changes should be made to it. A model of the queuing system under study must be constructed in this kind of analysis and the results of queuing theory are required to obtain the characteristics of the model and to assess the effects of changes, such as the addition of an extra server or a reduction in mean service time.

Perhaps the most important general fact emerging from the theory is that the degree of congestion in a queuing system (measured by mean wait in the queue or mean queue length) is very much dependent on the amount of irregularity in the system. Thus congestion depends not just on mean rates at which customers arrive and are served and may be reduced without altering mean rates by regularizing arrivals or service times, or both where this can be achieved.

1.2 Structure of a Queuing System

The major components of any Queuing system are shown in figure. We discuss these components below.

- Calling Population (input source)
- Queuing process
- Queue discipline
- Service process (Mechanism)

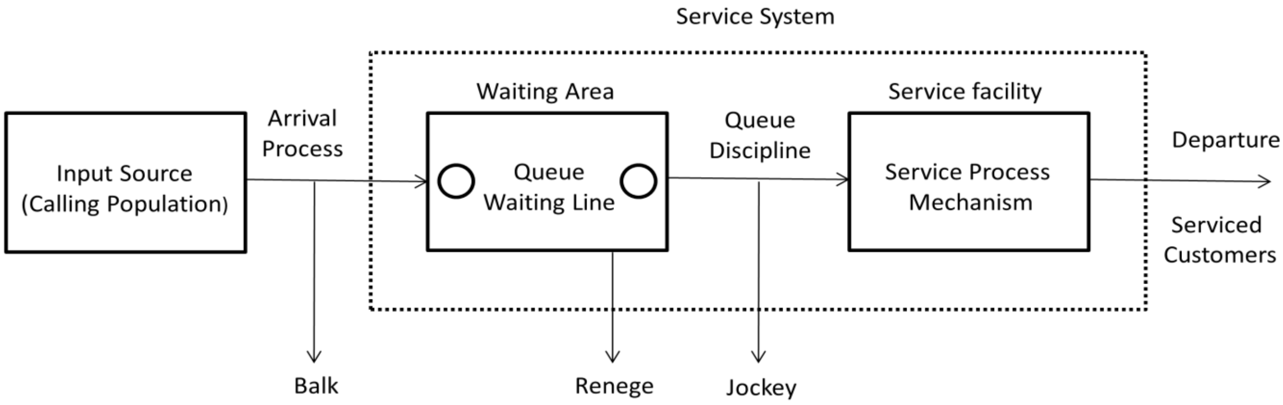

Fig. 5

Potential customers who arrive to the queuing system are referred as ***calling population***, also known as customer (input) source. The manner in which customer arrives at the service facility, individually or in batches at scheduled or unscheduled time is called the ***arrival process***. The customer's entry into the queuing system depends upon the queue condition. Customers, from a queue, are selected for service according to certain rules known as ***Queue discipline***. A service facility may be without server (self service), or may include one or more servers operating either in series or in parallel. The rate at which service is rendered is known as ***service process***. After the service rendered, the customer leaves the system.

If the server is idle at the time of customer's arrival, then the customer is served immediately, otherwise the customer is asked to join a queue (waiting line), which may have single, multiple or even priority lines.

Lecture

2

2.1 Calling population:

The arrival or inputs to the system are characterized by:

- Size of calling population
- Behavior of arrival
- Pattern of arrival at system

The calling population need not be homogeneous and may consist of several subpopulations. For example, patients arriving at the OPD of hospital are usually of three categories: walk-in patients, patients with appointments and emergency patients. Each patient class places different demands on service facilities, but the waiting expectation of each category differ significantly.

2.1.1 Size of calling population

- Limited or finite :

  If probability of an arrival depends on the number of customers already in the system (in service + in queue), the calling population is called limited or finite. examples of finite population are queuing system that have limited access for service, such as (I) machine operator responsible only to handle 5 machines (say), (ii) salesman responsible to handle limited number of customers, etc.

- Unlimited or infinite :

  If probability of an arrival does not depends on the number of customers already in the system (in service + in queue), the calling population is called unlimited or infinite. Examples of infinite population are those open to general public, such as banks, supermarkets, petrol pump, ticket counter, cinema halls, restaurants, etc. because in such a queuing system customers already present do not decreases potential of others in the population to enter the system.

2.1.2 Behavior of arrival

- Patient customer:
  If the customer, on arriving at the service system waits in the queue until served and does not switch between waiting line, he is called a patient customer. Machines arrived at the maintenance shop are example of patient customers.
- Impatient customer:
  The customer, who waits for a certain time in queue and leaves the service system without getting service due to certain reasons, is called an impatient customer. For example, a customer who has just arrived at a grocery shop and finds that the salesman are busy in serving the customers already in the system, will either wait for service till his patience is exhausted or estimates that his waiting time may be excessive and so leaves immediately to seek service elsewhere.
- Balking:
  Customers do not join the queue either by seeing the number of customers already in service system or by estimating the excessive waiting time for the desire service.
- Reneging:
  Customers, after joining the queue, wait for sometime in the queue but leave before being served on account of the certain reasons.
- Jockeying:
  Customers move from one queue to another hopping to receive service more quickly (a commonsense at a railway booking window).

2.1.3 Pattern of arrival at system

Customers may arrive in batches (such as the family at restaurant) or individually (such as train at platform). These customers may arrive at a service facility either on scheduled time or on unscheduled time (without informing). The arrival process of customers to the service system is classified into two categories:

- Static Arrival Process:
  The static arrival process is controlled by the nature of arrival rate. In random arrivals the time of arrival is a random variable and therefore need to understand the average and frequency distribution of the time. In both the cases, the arrival process can be described either by the average arrival rate or by the average interval time.
- Dynamic Arrival Process:
  The dynamic arrival process is controlled by both the service facility and the customers. The service facility adjusts its capability by either varying manpower at different timing

of service, varying service charges (telephone call charges at different hours of the day) at different timings, or allowing entry with appointments to match changes requires in the service intensity.

2.2 Queuing process:

The queuing process refers to the number of queues - single, multiple or priority queues and their lengths. The type of queue depends on the layout of service mechanism and the length (or size) of a queue depends upon operational situations such as physical space, legal restrictions, and attitude of the customers.

2.2.1 Finite Source Queue:

In certain cases, a service system is unable to accommodate more than the required number of customers at a time. No further customers are allowed to enter until more space is made available to accommodate new customers. Such type of situations is referred to as finite (or limited) source queue. Examples of finite source queues are cinema halls, restaurants, etc.

2.2.2 Infinite Source Queue:

If a service system is able to accommodate any number of customers at a time, then it is referred to as infinite (or unlimited) source queue. For example, in a sales department where the customer orders are received, there is no restriction on the number of orders that can come in.

On arriving at a system, if customers find long queue in front of a service facility, then they often do not enter the service system inspite additional waiting space available. The queue length in such cases depends upon *the attitude of customers*. For example, when a motorist finds that there are many vehicles waiting at petrol station, in most of the cases, he does not stop at this station and seeks service elsewhere.

In some finite source queuing systems, no queue is allowed to form. For example, when a parking space cannot accommodate additional incoming vehicles, the motorists are diverted elsewhere.

Multiple queues at a service facility can also be finite or infinite. But this has certain advantages such as:

- Division of manpower is possible.
- Customer has the option of joining any queue.
- Balking behavior of the customers can be controlled.

2.3 Queue discipline:

The queue discipline is the order (or manner) in which customers from the queue are selected for service. There are a number of ways in which customers in the queue are served. Some of these are:

2.3.1 Static Queue Disciplines

These are based on the individual customer's status in the queue. Few of such queue disciplines are:

- First Come First Served (FCRS):

  If customers are served in the order of their arrival, then this is known as the First Come First Served (FCRS) service discipline. Prepaid taxi queue at airports where a taxi is engaged on a 'first-come, first-served' basis is an example of this discipline.

- Last Come First Served (LCRS):

  In this discipline is mostly practiced in cargo handling where the last item loaded is removed first because it reduces handling and transportation cost, and in production process where items arrive at a workplace are stacked one on top of the other and item that is the last one to have arrived for service is processed first.

2.3.2 Dynamic Queue Disciplines

These are based on the Individual customer attributes in the queue. Few of such queue disciplines are:

- Service in random order (SIRO):

  Under this rule customers are selected for service it random, irrespective of their arrivals in the service system.

- Priority service:

  Under this rule customers are grouped in priority classes on the basis of some attributes such as service time or urgency. The FCFS rule is used within each class to provide service. The payment of telephone or electricity bills by cheque or cash are examples of this discipline.

- Pre-emptive priority (or Emergency):

  Under this rule, an important customer is allowed to enter into the service immediately after entering into the system, even if a customer with lower Priority is already in service. That is, lower priority customer's service is interrupted (preempted) to start the service for such a customer. This interrupted service is resumed after the priority customer is served.

- Non-pre-emptive priority:
  In this case an important customer is allowed to go ahead in the queue, but the service has started immediately on completion of the current service.

2.4 Service process:

The service mechanism (or process) is concerned with the manner in which customers are served and leave the service system. It is characterized by:

- The arrangement (or capacity) of service facilities
- The distribution of service times
- Server’s behavior
- Management policies

Lecture

3

3.1 Performance Measure of Queuing System:

The performance measures (operating characteristics) for the evaluation of the performance of an existing queuing system, and for designing a new system in terms of the level of service a customer receives as well as the proper utilization of the service facilities are listed as bellow:

Average (or expected) time spent by a customer in the queue and system

W_q : Average time an arriving customer has to wait in queue before being served.

W_s : Average time an arriving customer spends in the system, including waiting and service.

Average (or expected) number customer in the queue and system

L_q : Average number of customers waiting for service in the queue (queue length).

L_s : Average number of customers in the system (either waiting fo service in the queue or being served).

Value of time both for customers and servers

P_w : Probability that an arriving customer has to wait before being served (also called blocking probability)

P_n : Probability of n customers waiting in the queuing system.

P_d : Probability that an arriving customer is not allows to enter in the queuing system, i.e. system capacity is full.

$\rho = \frac{\lambda}{\mu}$: Percentage of time a server is busy serving customers, i.e. the system utilization.

Average cost required to operate the queue system

- Average cost required to operate the system per unit of time?
- Number of servers required to achieve cost effectiveness?

3.2 Notations

n	: Number of customers in the system (waiting and in service)
P_n	: Probability of n customers in system
λ	: Average customer arrival rate or average number of arrivals per unit time in queuing system
μ	: Average service rate or average number of customers served per unit time
ρ	$= \frac{\lambda}{\mu} = \frac{\text{Average service completion}(1/\mu)}{\text{Average interarrival timr }(1/\lambda)}$
P_0	: Probability of no customer in system
s	: Number of server
N	: Maximum number of customers allowed in the system
L_s	: Average number of customers in the system(waiting and in service)
L_q	: Average number of customers in the queue (queue length)
W_s	: Average waiting time in system (waiting and in service)
W_q	: Average waiting time in queue
P_w	: Probability that an arriving customer has to wait (system is busy)

3.3 Relation between Performance Measures

The following basic relationships holed for all infinite source queuing models.

$$L_s = \sum_{n=0}^{\infty} nP_n \qquad \text{and} \qquad L_q = \sum_{n=s}^{\infty} (n-s)P_n$$

The general relation among various performance measures is as follow:

I. Average number of customers in the system is equal to the average number of customer in queue plus average number of customers being served per unit time.

$$L_s = L_q + \frac{\lambda}{\mu}$$

The value of $\frac{\lambda}{\mu}$ is true for a single server finite source queuing model.

II. Average waiting time for a customer in the queue

$$W_q = \frac{L_s}{\lambda}$$

III. Average waiting time for a customer in the system including average service time

$$W_s = W_q + \frac{1}{\mu}$$

IV. Probability of being in the system longer than t given by:

$$P(T > t) = e^{-(\mu-\lambda)\lambda t} \quad \text{and} \quad p(T \leq t) = 1 - P(T > t)$$

Where T = time spent in the system

t = specified time period

e = 2.718

V. Probability of only waiting for service longer than time t is given by :

$$P(T > t) = \frac{\lambda}{\mu} e^{-(\mu-\lambda)t}$$

VI. Probability of exactly n customers in the system is given by:

$$P_n = P_0 \left(\frac{\lambda}{\mu}\right)^n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

VII. Probability that the number of customers in the system, n exceeds a given number, r is given by:

$$P(n > r) = \left(\frac{\lambda}{\mu}\right)^{r+1}$$

The general relationships among various performance measures are:

a. $L_s = \lambda W_s$
b. $W_s = W_q + \frac{1}{\mu} = \frac{1}{\lambda} L_s$
c. $L_q = L_s - \frac{\lambda}{\mu} = \lambda W_q$
d. $W_q = W_s - \frac{1}{\mu} = \frac{1}{\lambda} L_q$

3.4 Classification of queuing models

Queuing theory models are classified by using special notations describe initially by D.G. Kendall in the form (a/b/c). Later A.M. lee added the symbol d and e to Kendall's notation. In the literature of queuing theory, the standard format used to describe queuing models is as follows:

$$\{ (a/b/c) : (d/e) \}$$

Where,

a = arrivals distribution

b = service time distribution

c = number of servers (service cannel)

d = capacity of the system

e = queue discipline

In place of notation a and b, the following descriptive notation are used for the arrival and service time distribution:

M = Monrovian inter arrival time or service time distribution.

D = deterministic (constant) inter arrival time or service time.

G = general distribution of service time. i.e. no assumption is made about the type of distribution with mean and variance.

GI = general probability distribution – normal or uniform for inter arrival time

E_k = Erlang-k distribution for inter arrival or service time with parameter k (if k=1, Erlang is equivalent to exponential and if k=∞, Erlang is equivalent to deterministic).

Lecture 4

4.1 Model 1 - {(M/M/1): (∞/FCFS)}

4.1.1 Model 1: (MM/1): (FCFS)) Exponential Service - Unlimited Queue

This model based on certain assumptions about the queuing system:

- Arrivals are described by Poisson probability distribution and come from an infinite calling population.
- Single waiting line and each arrival waits to be served regardless of the queue (i.e. no limit on queue length - infinite capacity) and that there is on Balking or reneging.
- Queue discipline is “first come - first serve”.
- Single server or channel and service times follow exponential distribution.
- Customer’s arrival is independent but the arrival rate (average number of arrivals) does not change over time.
- The average service rate is more than the average arrival rate.

The following events (possibilities) may occur during the small interval of time, Δt just before time t.

- The system is in state n (number of customers) at time t and no arrival and no departure.
- The system is in state $n+1$(number of customer) and no arrival and one departure.
- The system is in state $n-1$ (number of customers) and one arrival and no departure.

Figure 6 illustrates the process of determining P_n (probability of n customers in the system at time t) when customers are either waiting or receiving service at each state. Customers may arrival or left by the completion of the leading customer's service.

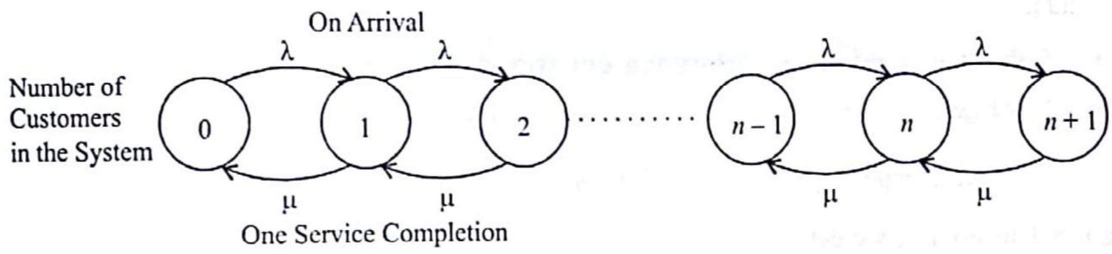

Fig.6

4.1.2 Steps for the solution:

1. Obtain system of differential difference equations.
2. Obtain the system of steady state equation.
3. Solve the system of difference equation.
4. Obtain probability density function of waiting time excluding service time distribution.
5. Calculate the busy period distribution.

4.2 Performance measure for model I

- Expected number of customer in the system (customer in line + customer being served)

$$L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

- Expected number of customer waiting in the queue (queue length)

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

- Expected waiting time for a customer in the queue.

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{L_q}{\lambda}$$

- Expected waiting time for a customer in the system (waiting + service).

$$W_s = \frac{L_s}{\lambda} = \frac{1}{\mu-\lambda}$$

- The variance (fluctuation) of the queue length.

$$Var(n) = \frac{\rho}{(1-\rho)^2} = \frac{\lambda\mu}{(\mu-\lambda)^2}$$

- Probability that the queue is non- empty.

$$P(n>1) = \left(\frac{\lambda}{\mu}\right)^2$$

- Probability that the number of customer, n in the system exceed a given number k.

$$P(n \geq k) = \left(\frac{\lambda}{\mu}\right)^k$$

- Expected length of non-empty queue.

$$L = \frac{\mu}{\mu - \lambda}$$

4.3 Examples 1

A television repairman find that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs the sets in the order in which they came in. and if the arrival of seat follows a Poisson distribution with an approximate average rate of 10 per 8 - hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

Solution:

From the data of the problem we have, $\lambda = \frac{10}{8}$ set per hour ; $\mu = \frac{60}{30} = 2$ sets per hour

(i) Expected ideal time of repairman each day

Since number of hours for which he repairman remains busy in 8-hour day is given by:

$$(8)\left(\frac{\lambda}{\mu}\right) = 5 \text{ hours}$$

Therefore, the ideal time for a repairman in 8-hour day will be $(8 - 5) = 3$ hours

(ii) Expected (or average) number of TV sets in the system

$$L_s = \frac{\rho}{1 - \rho} = \frac{\lambda}{\mu - \lambda} = \frac{5}{3} = 2(\text{approx}) \text{ TV sets}$$

Lecture

5

5.1 Example 1

In a railway marshalling yard, good trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time (the time taken to hump a train) distribution is also exponential with an average of 36 minutes. Calculate:

(i) Expected queue size. (line light)
(ii) Probability that the queue size exceeds 10

If the input of trains increases to an average of 33 per day, what will be the change in (i) and (ii)?

Solution:

From the data of the problem we have,

$\lambda = \frac{(30)(24)}{60} = \frac{1}{48}\text{train per mimute}\ ; \mu = \frac{1}{36}\ \text{train per minute}$

The traffic intensity then is, $\rho = \frac{\lambda}{\mu} = \frac{36}{48} = 0.75$

(i) Expected queue size:

$$L_s = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = 3\ \text{trains}$$

(ii) Probability that the queue size exceeds 10

$$P(n \geq 10) = \left(\frac{\lambda}{\mu}\right)^{10} = (0.75)^{10} = 0.06$$

If the input increase to 33 trains per day, the we have $\lambda = \frac{(33)(24)}{60} = \frac{11}{480}\text{train per minute}$

And $\mu = \frac{1}{36}$ train per minute.

Thus, traffic intensity, $\rho = \frac{\lambda}{\mu} = \frac{(11)(36)}{480} = 0.83$

Hence, recalculating the value of (I), (ii)

$L_s = \frac{\rho}{1-\rho} = \frac{0.83}{1-0.83} = 5 \text{ trains(approx)}$, and

$P(n \geq 10) = \left(\frac{\lambda}{\mu}\right)^{10} = (0.83)^{10} = 0.2$(approx)

5.2 Example 2

Arrivals at telephone booth are considered to be Poisson with an average time of 10 minute between one arrival and the next. The length of phone calls is assumed to be distributed exponentially, with a mean of 3 minutes.

(a) What is the probability that a person arriving at the booth will have to wait?
(b) The telephone department will install a second both when convinced that an arrival would expect waiting for at least 3 minute for a phone call. By how much should the flow of arrival increase in order to justify a second booth?
(c) What is the average length of the queue that forms time to time?
(d) What is the probability that it will take a customer more than 10 minutes altogether to wait for the phone and complete his call?

Solution:

From the data of the problem we have,

$\lambda = \frac{1}{10} = 0.10 \text{ person per mimute}$; $\mu = \frac{1}{3} = 0.33 \text{ person per minute}$

(a) Probability that a person has to wait at the booth.

$$P(n > 0) = 1 - P_0 = \frac{\lambda}{\mu} = 0.3$$

(b) The installation of second booth will be justified only if the arrival rate is more than the waiting time.
Let λ' be the increased arrival rate. Then the expected waiting time in the queue will be

$$W_q = \frac{\lambda'}{\mu(\mu - \lambda')}$$

$$3 = \frac{\lambda'}{0.33(0.33 - \lambda')}$$

$$\lambda' = 0.16$$

Where $W_q = 3$ (given) and $\lambda = \lambda'$ (say) for second booth. Hence the increase in the arrival rate is $0.16 - 0.10 = 0.06$ arrivals per minute.

(c) Average length of non-empty queue:

$$L = \frac{\mu}{\mu - \lambda} = 2 \text{ customers (approx)}$$

(d) Probability of waiting for 10 minutes or more is given by:

$$P(t \geq 10) = \int_{10}^{\infty} \frac{\lambda}{\mu}(\mu - \lambda)e^{-(\mu-\lambda)t}dt$$

$$P(t \geq 10) = \int_{10}^{\infty} (0.3)(0.23)e^{-(0.23)t}dt$$

$$= 0.069\left[\frac{e^{-(0.23)t}}{-0.23}\right]_{10}^{\infty}$$

$$= 0.03$$

5.3 Example 3

A warehouse has only one loading dock manned by a three person crew. Trucks arrive at the loading dock at an average rate of 4 trucks per hour and the arrival rate is Poisson distributed. The loading of a truck take 10 minutes on an average and can be assumed to be exponentially distributed. The operating cost of a truck is Rs 20 per hour and the member of the loading crew are paid Rs 6 each per hour. Would you advise the truck owner to add another crew of three persons?

Solution:

From the data of the problem we have,

$\lambda = 4$ per hour ; $\mu = 6$ per hour

For existing crew

$$\text{Total hourly cost} = \text{loading crew cost} + \text{cost per waiting time}$$

$$= \{(\text{number of leaders}) \times (\text{hourly wage rate})\}$$

$$+ \{(\text{expected waiting time per truck } W_s)(\text{expected arrival per hour } \lambda)$$

$$(\text{hourly waiting cost})\}$$

$$= (6 \times 3) + \left(\frac{1}{6-4}\right)(4 \times 20)$$

$$= \text{Rs.} 58 \ \text{ per hour}$$

After proposed crew

$$\text{Total hourly cost } = (6 \times 6) + \left(\frac{1}{12-4}\right)(4 \times 20)$$

$$= \text{Rs.} 46 \ \text{ per hour}$$

Since the total hourly cost after the addition of another crew of 3 person is less than the exiting cost, therefore the truck owner must add a crew of another 3 leaders.

5.4 Example 4

A road transport company has one reservation clerk on duty at a time. He handles information of bus schedules and makes reservations. Customers arrive at a rate of 8 per hour and the clerk can, on an average, service 12 customer per hour. After stating your assumption, answer the following:

(a) What is the average, number of customer waiting for the service of the clerk?
(b) What is the average time a customer has to wait before being served?
(c) The management is contemplating to install computer system for handling information and reservations. This is an expected to reduce the service time from 5 to3 minutes. The additional cost of having the new system works out to Rs 50 per day. If the cost of goodwill of having to wait is estimated to be 12 paisa, per minute spent waiting, before being served, should the company install the computer system? Assume an 8 hour working day.

Solution:

From the data of the problem we have, $\lambda = 8 \text{ per hour}$; $\mu = 12 \ \text{ per hour}$

(a) The average number of customers waiting for the service in the system are

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{8}{12-8} = 2 \text{ customers}$$

(b) The average time spent by a customer in the system before being served is

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{12 - 8} = \frac{1}{4}\text{hour} = 15\text{ minute}$$

The average waiting time for a customer in the queue is

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{8}{12(12 - 8)} = \frac{1}{6}\text{hour} = 10\text{ minute}$$

(c) The additional cost of Rs. 50 per day should be compared with the difference in the goodwill cost of customers with one existing system and a computer system, before installing the computer system.

The average cost for a customer's waiting time ($W_s = 15$ minute) in the system, is

$$0.12 \times 15 = \text{Rs }1.80$$

Also there are 8 arrivals per hour or in 8 hours $64(= 8 \times 8)$ customers request service at a total goodwill cost of $1.80 \times 64 = \text{Rs }115.20$

By installing computer system, the computer will increase a clerk's service rate up to $\mu = 20$ customers per hour (3 customers per minute).

Thus the average time spent by a customer in the system would be

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{20 - 8} = \frac{1}{12}\text{hour} = 5\text{ minute}$$

And the average daily customer queuing (or goodwill) cost would be reduce to $64 \times (0.12 \times 5) = \text{Rs.} 38.40$

An additional cost of having the computer would be Rs. 50 per day

Thus the average total daily cost would be

$$\text{TC} = \text{computer cost} + \text{goodwill cost} = 50 + 38.40 = \text{Rs.} 88.40$$

This cost is less than the existing goodwill loss cost and gives a net saving of $Rs. (115.20 - 88.40) = \text{Rs.} 25.80$

Hence company can install a computer.

Lecture 6

6.1 Exercise 1

At what average rate must a clerk at a supermarket work in order to ensure a probability of 0.90 so that the customer will not have to wait longer than 12 minute? It is assumed that there is only one counter at which customer arrive in a Poisson fashion at an average rate of 15 per hour. The length of service by the clerk has an exponential distribution.

Answer:

$$\lambda = \frac{15}{60} = \frac{1}{4} \quad ; \; \mu = ?$$

$$P(\text{waiting } \geq 12) = \int_{12}^{\infty} \frac{\lambda}{\mu}(\mu - \lambda)e^{-(\mu-\lambda)t}dt = 0.10(\text{given})$$

$$\frac{1}{\mu} = 2.48 \text{ minute/service}$$

6.2 Exercise 2

Consider a self-service store with one cashier. Assume Poisson arrivals and exponential service times. Suppose that on average 9 customers arrive every 5 minutes and that the cashier can serve 10 in 5 minutes. Find:

(a) Average number of customer queuing for service
(b) Probability of having more than 10 customer in the system , and
(c) Probability that a customer has to queue for more than 2 minutes.

If the service can be speed up to 12 in 5 minutes by using a different cash register, what will be the effect of this on the quantities (a), (b) and (c).

Answer:

Case 1:

$$\lambda = \frac{9}{5} \quad ; \ \mu = \frac{10}{5}$$
$$L_s = \frac{\rho}{1-\rho} = \frac{0.9}{1-0.9} = 9 \text{ customers}$$
$$P(n \geq 10) = \left(\frac{\lambda}{\mu}\right)^{10} = (0.9)^{10}$$
$$P(\text{waiting } \geq 2) = \int_2^\infty \frac{\lambda}{\mu}(\mu - \lambda)e^{-(\mu-\lambda)t}dt = 0.67$$

Case 2:

$$\lambda = \frac{9}{5} \quad ; \ \mu = \frac{12}{5}$$
$$L_s = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = 3 \text{ customers}$$
$$P(n \geq 10) = \left(\frac{\lambda}{\mu}\right)^{10} = (0.75)^{10}$$
$$P(\text{waiting } \geq 2) = \int_2^\infty \frac{\lambda}{\mu}(\mu - \lambda)e^{-(\mu-\lambda)t}dt = 0.30$$

6.3 Exercise 3

Customer arrives at a box office window, being manned by a single individual, according to a Poisson input process, with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 seconds. Find the average waiting time of a customer. Also determine the average number of customer in the system and average queue length.

Answer:

$$\lambda = \frac{30}{60} \quad ; \ \mu = \frac{60}{90}$$
$$W_s = 4.5 \text{ minute per customer}$$
$$L_s = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = \ 3 \text{ customers}$$
$$L_q = 0.25$$

6.4 Exercise 4

The mean rate of arrival of planes at airport during the peak period is 20 per hour, and the actual number of arrivals in any hour follows a Poisson distribution. The airport can land 60 planes per hour on an average, in good weather, and 30 planes per hour in bad weather. The actual number landing in any hour follows a Poisson distribution, with these respective averages. When there is congestion, the planes are forced to fly over the field in the stack awaiting the landing of other planes that arrived earlier.

(a) How many planes would be flying over the field in the stack on an average, in good weather conditions and in bad weather conditions?

(b) How long would a plane be in the stack and in the process of landing in good and in bad weather?

Answer:

$$\lambda = 20 \quad ; \quad \mu = \begin{cases} 60\,; in\ good\ weather \\ 30\,;\ \ in\ bad\ weather \end{cases}$$

$$L_s = \begin{cases} \frac{1}{6}\,; in\ good\ weather \\ \frac{4}{3}\,;\ \ in\ bad\ weather \end{cases}$$

$$L_q = \begin{cases} \frac{1}{40}\,; in\ good\ weather \\ \frac{1}{10}\,;\ \ in\ bad\ weather \end{cases}$$

Lecture

7

7.1 Exercise 1

A repair shop, attended by a single mechanic, has an average of four customers an hour who bring small appliances for repair. The mechanic inspects them for defects and for this he takes six minutes on an average. Arrivals are Poisson and service rate has an exponential distribution. You are required to:

(a) Find the proportion of time during which there is no customer in the shop.
(b) Find the probability of finding at least one customer in the shop.
(c) Calculate is the average number of customer in the system?
(d) Find the average spent by a customer in the shop including service.

Answer:

$$\lambda = 4 \; per \; hour \quad ; \quad \mu = 10 \; per \; hour$$

$$P_0 = 1 - \rho = 0.6$$

$$P(n \geq 1) = 0.4$$

$$L_s = \frac{\rho}{1-\rho} = \frac{0.4}{1-0.4} = \frac{2}{3}$$

$$W_s = 10 \text{ minutes}$$

7.2 Exercise 2

In a bank, cheques are cashed at a single ' teller' counter customer arrive at the counter in a Poisson manner at an average rate of 30 customer per hour, the teller takes, on an average, a minute and a half to cash a cheque. The service time has shown to be exponentially distributed.

(a) Calculate the percentage of time the teller is busy.
(b) Calculate the average time a customer is expected to wait.

Answer:

$\lambda = 30\, per\ hour$; $\mu = 40\, per\ hour$

$P_0 = 1 - \rho = 0.25$

Busy period $= 1 - P_0 = 0.75$

$W_s = 6$ minutes

7.3 Exercise 3

In a tool crib manned by a single assistant, operators arrive at the tool crib the rate of 10 per hour. Each operator needs 3 minutes, on an average, to be served. Find out the loss of production due to the time lost in waiting for an operator in a shift of 8 hours, if the rate of production is 100 units per shift.

Answer:

$\lambda = 10\, per\ hour$; $\mu = 20\, per\ hour$

$W_q = 3$ minutes; Avarage waiting time oer shift is: $\frac{8}{20} = \frac{2}{5}$ hours

Loss of production due to waiting $= \left(\frac{2}{5}\right) \times \left(\frac{100}{8}\right) = 5$ units

Lecture 8

8.1 Exercise 1

Trucks arrive at a factory for collecting finished goods that are supposed to be transported to distant markets. As and when they come they are required to join a waiting line and are served on first-come, first-served basis. Trucks arrive at the rate of 10 per hour whereas the loading rate is 15 per hour. It is also given that arrivals are Poisson and loading is exponentially distributed. Transporters have complained that their trucks have to wait for nearly 1 hour at the plant. Examine the probability that loaders are idle in the above problem.

Answer:

$\lambda = 10\, per\ hour \ \ ; \ \mu = 15\, per\ hour$

$L_q = 8 \text{ minutes}$

Idle time $P_0 = 1 - \rho = 33.33\%$

8.2 Exercise 2

On an average 96 patients per 24 hour-days require the service of an emergency clinic. Also on an average patient require 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it cost the clinic Rs100 per patient treated to obtain an average servicing time of 10 minutes, and that each minute of decrease in this average time would cost Rs 10 per patient treated, how much would have to be budgeted by the clinic to decrease the average size of the queue from 4/3 patients to 1/2 patient.

Answer:

$\lambda = 10\, per\ hour \ \ ; \ \mu = 15\, per\ hour$

$L_q = \frac{4}{3}$ minutes. But if it is change from $\frac{4}{3}\ to\ \frac{1}{2}$, then new value of μ will be

$$L_q = \frac{\lambda^2}{\mu'(\mu' - \lambda)}$$

$$\frac{1}{2} = \frac{(96)^2}{\mu(\mu - 96)}$$

$\mu' = 192 \text{ patients/day}$

Thus average rate of treatment required is $\frac{1}{\mu'} = (60) \times \left(\frac{24}{192}\right) = 75 \text{ minutes;}$ decrease in average time of treatment required is $(10 - 7.5) = 2.5$ minutes. Revised budget per patient $= \text{Rs.}(100 + 2.5 \times 10) = Rs. 125.$

8.3 Exercise 3

In a service department manned by one server, on an average one customer arrives every 10 minutes. It has been found out that each customer requires 6 minutes to be served. Find out:

(a) Average queue length
(b) Average time spent in the system
(c) Probability that there would be two customers in the queue.

Answer:

$\lambda = 6 \; per \; hour \quad ; \quad \mu = 10 \; per \; hour$

$L_q = 0.9 \text{ customers}$

$W_s = 15 \text{ minutes}$

$$P(n \geq 2) = \left(\frac{\lambda}{\mu}\right)^2 \left(1 - \frac{\lambda}{\mu}\right) = (0.6)^2(0.4) = 0.144$$

Lecture 9

9.1 Model 2 - {(M/M/1): (∞/SIRO)}

This model is identical to Model 1 with the only difference in queue discipline. Since the derivation of P_n is independent of any specific queue discipline, therefore even in this model we have,

$$P_n = (1-\rho)\rho^n \; ; n = 1,2,\ldots$$

Consequently, other result will also remain unchanged as long as P_n remains unchanged.

9.2 Model 3 - {(M/M/1): (N/FCFS)}

9.2.1 Model 3 - {(M/M/1): (N/FCFS)} Exponential Service – Finite (or limited) Queue

This model is also based on all assumption of model 1, except a limit on the capacity of the system to accommodate only N customers. This implies that once line reached its maximum length of N customers, no additional customer will be allowed to enter into the system.

A finite queue may arise due to physical constrains such as emergency room in hospital; one man barber shop with certain number of chairs for waiting customers, etc.

The difference equation derived in model 1 will also be the same for this model as long as $n < N$. the systems of steady state difference equation for this model are

$$\lambda P_0 = \mu P_1 \qquad ; n = 0$$

$$(\lambda+\mu)P_n = \lambda P_{n-1} + \mu P_{n+1} \qquad ; n = 1,2,\ldots N-1$$

$$\lambda P_{N-1} = \mu P_N \qquad ; n = N$$

In this case the service rate dose not has to exceed arrival rate $(\mu > \lambda)$ in order to obtain steady state condition.

Using, the usual producer, from the first two difference equation, the probability of a customer in the system for $n = 1,2,\ldots,N$ is obtained as follow:

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \,; n \leq N$$

To obtain the value of P_0, use the fact that ;

$$1 = \sum_{n=0}^{N} P_n = \sum_{n=0}^{N} \left(\frac{\lambda}{\mu}\right)^n P_0$$

$$= P_0 \sum_{n=0}^{N} \left(\frac{\lambda}{\mu}\right)^n$$

$$= P_0 \sum_{n=0}^{N} \rho^n$$

$$= P_0[1 + \rho + \rho^2 + \rho^3 + \cdots + \rho^N]$$

$$= P_0 \left[\frac{1-\rho^{N+1}}{1-\rho}\right]$$

Consequently,

$$P_0 = \left[\frac{1-\rho}{1-\rho^{N+1}}\right] \;;\; \rho \neq 1 \text{ and } \rho < 1$$

$$P_n = \begin{cases} \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\rho^n & ; n \leq N \,; \rho \neq 1 \\ \frac{1}{N+1} & ; \rho = 1 \end{cases}$$

The steady state solution in this case exists even for $\rho > 1$. This is due to the limited capacity of the system. If $\lambda < \mu$ and $N \to \infty$, then $P_n = (1-\rho)\rho^n$, which is the same as in model 1.

9.3 Performance Measure for Model III

- Expected number of customer in the system (customer in line + customer being served)

$$L_s = \begin{cases} \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}} & ; \rho \neq 1 \\ \frac{N}{2} & ; \rho = 1 \end{cases}$$

- Expected number of customer waiting in the queue (queue length)

$$L_q = L_s - \rho \ = \ L_s - \frac{\lambda(1-P_n)}{\mu}$$

- Expected waiting time for a customer in the queue.

$$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda(1-P_N)}$$

- Expected waiting time for a customer in the system (waiting + service).

$$W_s = \frac{L_s}{\lambda(1-P_N)} = \frac{L_q}{\lambda(1-P_N)} + \frac{1}{\mu}$$

- Potential customers lost (time for which the system is busy).

$$P_N = P_0 \, \rho^N$$

Effective arrival rate $\lambda_{\text{eff}} = \lambda(1-P_N)$

Effective traffic intensity $\rho_{\text{eff}} = \frac{\lambda_{\text{eff}}}{\mu}$

9.4 Examples 1

Consider a single server queuing system with Poisson input and exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum permissible calling units in the system, is two, derive the steady–state probability distribution of the number of calling units in the system, and the calculate the expected number in the system.

Solution:

From the data of the problem we have, $\lambda = 3$ units per hour ; $\mu = 4 \, units$ per hour, $N = 2$, then traffic intensity $\rho = 0.75$

The steady state probability distribution of the number of n customers (calling units) in the system is:

$$P_n = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\rho^n = \left(\frac{1-0.75}{1-(0.75)^{2+1}}\right)(0.75)^n = (0.43)(0.75)^n \;\; ; \rho \neq 1$$

$$P_0 = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\rho^0 = \left(\frac{1-0.75}{1-(0.75)^{2+1}}\right)(0.75)^0 = \frac{0.25}{1-(0.75)^3} = 0.431$$

The expected number of calling units in the system is given by:

$$L_s = \sum_{n=1}^{N} nP_n = \sum_{n=1}^{2} n(0.43)\,(0.75)^n$$

$$= (0.43)\textstyle\sum_{n=1}^{2} n\,(0.75)^n$$

$$= (0.43)\{(0.75) + 2(0.75)^2\}$$

$$= 0.81$$

Lecture

10

10.1 Exercise 1

If in a period of 2 hours, in a day (8 to 10 a.m.), trains arrive at the yard every 20 minutes but the service time continuous to remain 36 minutes, the calculate, for this period:

(a) The probability that the yard is empty and
(b) The average number of trains in the system, on the assumption that the line capacity of the yard is only limited to 4 trains.

Answer:

$$P_0 = 0.04$$

$$L_s = 2.9 \approx 3$$

10.2 Exercise 2

At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while the other is given a signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. Also find the average waiting time of a new train arriving at the yard.

Answer:

$$P_0 = 0.53$$

$$L_s = 0.74$$

$$L_q = 0.24$$

$$W_q = 0.04$$

10.3 Exercise 3

Patients arrive at a clinic according to a Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. The examination time per patient is exponential with mean rate of 20 per hour.

(i) Find the effective arrival rate at the clinic.

(ii) What is the probability that an arriving patient will not wait? Will he find a vacant seat in the room?

(iii) What is the expected waiting time until a patient is discharged from the clinic?

Answer:

$\lambda = \frac{30}{60}$,

$\mu = \frac{20}{60}$,

$\rho = \frac{2}{3}$,

$N = 14$,

Then find P_0 , P_n , W_s

10.4 Exercise 4

Assume that goods trains are coming in a yard at the rate of 30 trains per day and suppose that the interarrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines, one of which is reserved for shunting purpose). Calculate the probability that the yard is empty and find the average queue length.

Answer:

$\lambda = \frac{30}{(60)(24)} = \frac{1}{48}$,

$\mu = \frac{1}{768}$,

$\rho = 0.75$,

$P_0 = 0.28$,

$L_q = 1.55$

10.5 Exercise 5

A petrol station has single pump and space for not more than 3 cars (2 waiting, 1 being served). A car arriving when the space is filled to capacity goes elsewhere for petrol. Cars arrive according to a Poisson distribution at a mean rate of 1 every 8 minutes. Their service time has an exponential distribution with the mean of 4 minutes.

The owner has a opportunity of renting an adjacent piece of land, which would provide space for an additional car to wait (he could not built another pump). The rent would be Rs. 2000 per month. The expected net profit from each customer is Rs. 2 and the station is open 10 hours every day. Would it be profitable to rent the additional space?

Answer:

$$\lambda = \frac{30}{(60)(24)} = \frac{1}{48} \ ,$$

$$\mu = \frac{1}{36} \ ,$$

$$\rho = 0.75 \ \ ,$$

$$P_0 = 0.28 \ \ ,$$

$$L_s = 3 \text{ trains}$$

Lecture

11

11.1 National Eligibility Test: NET

Question 120 **Part C, June 2011**

Let $X(t)$ be the number of customers in the $M/M/1$queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. It is known that

$$\lim_{n\to\infty} P(X(t)-1) = \frac{1}{4}$$

Which of the following is true?

a) $\boldsymbol{\lim_{t\to\infty} E(X(t)=1) = \frac{1}{3}}$

b) $\lim_{t\to\infty} E(X(t)=1) = \frac{\lambda}{\mu}$

c) $\boldsymbol{\lim_{t\to\infty} Var(X(t)=1) = \frac{1}{9}}$

d) $\lim_{t\to\infty} Var(X(t)=1) = \left(\frac{\lambda}{\mu}\right)^2$

Question 120 **Part C, December 2011**

Let Q_n denote the length of queue at time n in a $M/M/1$ queue with arrival rate $\lambda > 0$, service rate $\mu > 0$ and $\rho = \frac{\lambda}{\mu}$. Which of the following is true?

a) If $\boldsymbol{\lambda < \mu}$, then $\boldsymbol{\lim_{n\to\infty} P(Q_n = k) = (1-\rho)\rho^k,\ k \geq 0}$

b) If $\lambda = \mu$, then $\lim_{n\to\infty} P(Q_n = k) = \frac{1}{2^{k+1}},\ k \geq 0$

c) If $\lambda > \mu$, then $\lim_{n\to\infty} P(Q_n = k) = (1-\rho)\rho^k,\ k \geq 0$

d) If $\boldsymbol{\lambda = \mu}$, then $\boldsymbol{\lim_{n\to\infty} P(Q_n = k) = 0,\ k \geq 0}$

Question 60 **Part B, June 2012**

Let $X(t)$ be the number of customers in the $M/M/1$queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. The process $X(t)$ is a

a) Poisson process with rate $\lambda - \mu$

b) Pure birth process with birth rate $\lambda - \mu$

c) Birth and death process with birth rate λ and death rate μ

d) Birth and death process with birth rate $\frac{1}{\lambda}$ and death rate $\frac{1}{\mu}$

Question 120 **Part C, June 2012**

In a system with a single server, suppose that the customer arrive at Poisson rate of 1 person every 12 minute and are serviced at the Poisson rate of 1 service every 8 minute. If the arrival rate is increased by 20%, then in the steady state

a) The increase in the average number of customers in the system is 2

b) The increase in the average number of customers in the system is 4

c) The increase in the average time spent by customers in the system is 16 minute

d) The increase in the average time spent by customers in the system is 24 minute

---

Question 60 **Part B, December 2012**

Men arrive in a queue according to a Poisson process with rate λ_1 and woman arrive in the same queue according to another Poisson process with rate λ_2 . The arrivals of man and woman are independent. The probability that the first arrival in the queue is a man, is

a) $\boldsymbol{\frac{\lambda_1}{\lambda_1+\lambda_2}}$

b) $\frac{\lambda_2}{\lambda_1+\lambda_2}$

c) $\frac{\lambda_1}{\lambda_2}$

d) $\frac{\lambda_2}{\lambda_1}$

Question 118 **Part C, December 2013**

Consider a queuing model with one service counter. The arrival and service processes are Poisson with rate λ and μ, respectively. For $n = 0,1,2 \ldots$ and $\lambda < \mu$, let $p_n = P$ {at any point of time , there are n customers in the system} $= \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$.then, the average queue length is

a) $\frac{\lambda}{\mu-\lambda}$

b) $\frac{\lambda}{\mu(\mu-\lambda)}$

c) $\boldsymbol{\frac{\lambda^2}{\mu(\mu-\lambda)}}$

d) $\frac{\mu}{\mu-\lambda}$

Question 120 **Part C, June 2015**

Let $X(t)$ be the number of customers in the system at time t in an $M/M/C$ queuing model, with $C = 3$, arrival rate $\lambda > 0$ and service rate $\mu > 0$. which of the following is correct ?

a) $\{X(t)\}$ is a birth and death process with constant birt and deaths rates

b) If $\{X(t)\}$ has a stationary distribution, then $\lambda < 3\mu$

c) If $\lambda < 3\mu$, then the stationary distribution is a geometric distribution with parameter $\frac{\lambda}{3\mu}$

d) The number of customers undergoing service at time t is $\min\{X(t), 3\}$

Question 120 **Part C, December 2015**

Consider an M/M/1 queue with arrivals as a Poisson process at a rate of 8 per hour and a service time which is exponentially distributed at a rate of 6 minutes per customer. The waiting time of a customer in the queue

a) has a gamma distribution with p.d.f.

b) has distribution function given by

c) has mean 4 minutes.

d) has mean 24 minutes.

Question 60 **Part B, June 2016**

Customers arrive at an ice cream parlor according to a Poisson process with rate 2. Service time distribution has density function $f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0 & , x \le 0 \end{cases}$

Upon being served a customer may rejoin the queue with probability 0.4, independently of new arrivals; also a returning customer's service time is the same as that of a new arriving customer. Customers behave independently of each other. Let $X(t) =$ number of customers in the queue at time t. Which among the following is correct?

a) $\{X(t)\}$ grows without bound with probability 1.

b) $\{X(t)\}$ has stationary distribution given by $\pi_k = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^k, k = 0,1,\ldots$

c) $\{X(t)\}$ has stationary distribution given by $\pi_k = \left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^k, k = 0,1,\ldots$

d) $\{X(t)\}$ has stationary distribution given by $\pi_k = \left(\frac{4}{10}\right)\left(\frac{6}{10}\right)^k, k = 0,1,\ldots$

Question 60 **Part B, December 2016**

Consider an M/M/1 Queue with arrival rate λ and service rate μ with $\lambda < \mu$ What is the probability that no customer exited the system before time 5?

a) $\boldsymbol{\frac{\mu e^{-5\lambda} - \lambda e^{-5\mu}}{\mu - \lambda}}$

b) $e^{-5\lambda} - e^{-5\mu}$

c) $e^{-5\lambda} + (1 - e^{-5\lambda})\frac{e^{-5\mu}}{5\mu}$

d) $e^{-5\mu} + (1 - e^{-5\mu})\frac{e^{-5\lambda}}{5\lambda}$

Question 120 **Part C, December 2017**

Consider an $M/M/1$ queue with interarrival time having exponential distribution with mean $\frac{1}{\lambda}$ and service time having exponential distribution with mean $\frac{1}{\mu}$ which of the following are true?

a) if $0 < \lambda < \mu$ then the queue length has limiting distribution Poisson $\mu - \lambda$

b) if $0 < \mu < \lambda$ then the queue length has limiting distribution Poisson $\lambda - \mu$

c) if $0 < \lambda < \mu$ then the queue length has limiting distribution which is geometric

d) if $0 < \mu < \lambda$ then the queue length has limiting distribution which is geometric

Question 119 **Part C, June 2018**

Consider a single server model $M/M/1$ queue with arrival rate λ and service rate μ. Further assume that $\lambda < \mu$. Then, which of the following statements are true?

a) Queue length become 0 in infinitely many time interval with probability 1

b) Queue length become 0 in at most finitely many time interval with probability 1

c) Steady state exists for the queue.

d) $\lim_{t\to\infty} P(L_t > 0) = \frac{\lambda}{\mu}$, where L_t is the number of customer in system at time t

---

Reference Books:

- J. K. Sharma, “Operation Research – Theory and Application”, 4th Edition, MacMillan Publishers India Ltd.
- S. D. Sharma, “Operation Research – Theory, Methods and Application”, Kedar Nath Ram Nath.
- N. H. Shah, Ravi Gor, Hardik Soni, “Operation Research”, Prentice Hall of India.
- F. S. Hiller and G. J. Leiberman “Introduction to Operation Research”, Tata McGraw Hill.

Notes:

❖