U-2

Q-1 Check the optimality of soln for the following transportation problem using VAM. If the soln is not optimal, modify it.

Factory		Retail Shops R_1	R_2	R_3	R_4	R_5	Capacity
	F_1	1	9	13	36	51	50
	F_2	24	12	16	20	1	100
	F_3	14	35	1	23	26	150
		100	70	50	40	40	300

→ $\Sigma a_i = \Sigma b_j = 300$

→ This is balanced transportation problem

⇒ VAM :-

Factory		R_1	R_2	R_3	R_4	R_5		Penalties				
	F_1	1/50	9/	13/	36/	51/	50/0	8	8	8	-	-
	F_2	24/	12/60	16/	20/	1/40	100/60/0	11	4	8	8	-
	F_3	14/50	35/10	1/50	23/40	26/	150/100/50/0	13	13	9	9	9
		100/50/0	70/10/0	50/0	40/0	40/0	300					
		13	3	12	3	25						
	Penalties	13	3	12	3	-						
		13	3	-	3	-						
		10	23	-	3	-						
		14	-	-	23	-						

→ Total Cost = 50 + 720 + 40 + 700 + 350 + 50 + 920 = 2830 //

⇒ So IBFS is

	R_1	R_2	R_3	R_4	R_5
F_1	1/50	9	13	36	51
F_2	24	12/60	16	20	1/40
F_3	14/50	35/10	1/50	23/40	26

→ Test for optimality

	R_1	R_2	R_3	R_4	R_5	
F_1	(1)	[13/9]	25/13	26/36	40/51	$U_1 = 0$
F_2	33/24	(12)	38/16	20/20	(1)	$U_2 = 10$
F_3	(14)	(35)	(1)	(23)	26	$U_3 = -13$
	$V_1 = -1$	$V_2 = -22$	$V_3 = 12$	$V_4 = -10$	$V_5 = -11$	

→ Since we get one entry R_{21} as negative so present solution is not optimal.

→ Improvision of sol^n

	R_1	R_2	R_3	R_4	R_5
F_1	50-x •	• +x			
F_2	)	•			•
F_3	50+x •	• 10-x	•	•	

→ Let $x = \min\{50, 10\} = 10$

→ Improvised soln is

	R_1	R_2	R_3	R_4	R_5	
F_1	(1/40)	9/10	12/13	26/36	53/51	$U_1 = 0$
F_2	20/24	(12/60)	1/16	7/20	(1/40)	$U_2 = -3$
F_3	(14/60)	13/35	(1/50)	(23/40)	15/26	$U_3 = -13$
	$V_1 = -1$	$V_2 = -9$	$V_3 = 12$	$V_4 = -10$	$V_5 = 2$	

→ Since all $R_{ij} \geq 0$, so present soln is optimal

→ Minimum transportation Cost = 40 + 720 + 40 + 840 + 50 + 920
= 2,700 //

Q-2 Solve the following transportation problem to get the minimum transportation cost:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	90	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

→ Here $\Sigma a_i = \Sigma b_j = 34$

→ This is balanced transportation problem.

⇒ VAM:-

	D_1	D_2	D_3	D_4	Supply	Penalties			
S_1	(19/5)	30/	50/	(10/2)	7/2/0	9	9	40	40
S_2	70/	30/	(40/7)	(60/2)	9/2/0	10	20	20	20
S_3	40/	(8/8)	70/	(20/10)	18/10/0	12	20	50	-
Demand	5/0	8/0	7/0	18/10/0					
	21	22	10	10					
Penalties	21	-	10	10					
	-	-	10	10					
	-	-	10	50					

→ Total Cost = 95 + 20 + 280 + 120 + 64 + 200 = 779 //

→ So IBFS is

	D_1	D_2	D_3	D_4
S_1	19/5	30	50	10/2
S_2	70	30	40/2	60/2
S_3	40	8/8	70	20/10

→ Test for optimality

	D_1	D_2	D_3	D_4	
S_1	(19)	32/30	60/50	(10)	$U_1 = 0$
S_2	1/70	[-18/30]	(40)	(60)	$U_2 = -50$
S_3	11/40	(8)	70/70	(20)	$U_3 = -10$
	$V_1 = -19$	$V_2 = 2$	$V_3 = 10$	$V_4 = -10$	

→ Since we get one enty D_{22} as negative so present solution is not optimal.

→ Improvision of solution

	D_1	D_2	D_3	D_4
S_1	•			•
S_2		$+x$ *	•	• $2-x$
S_3		$8-x$ •		• $10+x$

→ Let $x = \min\{2, 8\} = 2$

→ Improvised Soln is

	D_1	D_2	D_3	D_4
S_1	19/5	30	50	10/2
S_2	70	30/2	40/7	60
S_3	40	8/6	70	20/12

→ Test for optimality

	D_1	D_2	D_3	D_4	
S_1	(19)	32/30	42/50	(10)	$U_1 = 0$
S_2	19/70	(30)	(7)	18/60	$U_2 = -32$
S_3	11/40	(8)	52/70	(20)	$U_3 = -10$
	$V_1 = -19$	$V_2 = 2$	$V_3 = -8$	$V_4 = -10$	

→ Since all $d_{ij} \geq 0$, so present soln is optimal

→ Minimum transportation Cost = 95+20+60+280+48+240 = 743//

Q3 Solve the following transportation problem for the minimum transportation cost:

	D_1	D_2	D_3	Supply
P_1	16	20	12	200
P_2	14	8	18	160
P_3	26	24	16	90
Demand	180	120	150	

→ $\Sigma a_i = \Sigma b_j = 450$

→ This is balanced transportation problem

⇒ VAM:-

	D_1	D_2	D_3	Supply	Penalties			
P_1	16/140	20/	12/60	200/140/0	4	4	4	16
P_2	14/40	8/120	18/	160/40/0	6	4	4	14
P_3	26/2	24/	16/90	90/0	8	10	-	-
Demand	180/40/0	120/0	150/60/0					
	2	12	4					
Penalties	2	-	4					
	2	-	6					
	2	-	-					

→ Total Cost: 2,240 + 720 + 560 + 960 + 1,440 = 5,920 //

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→ So IBFS is

	D_1	D_2	D_3
P_1	16/140	20	12/60
P_2	14/40	8/120	18
P_3	26	24	16/90

→ Test for Optimality:

	D_1	D_2	D_3	
P_1	(16)	10/20	(12)	$U_1 = 0$
P_2	(14)	(8)	8/18	$U_2 = 2$
P_3	6/26	10/24	(16)	$U_3 = -4$
	$V_1 = -16$	$V_2 = -10$	$V_3 = -12$	

→ Since all $d_{ij} \geq 0$
So final optimal soln is arrived.

	D_1	D_2	D_3
P_1	16/140	20	12/60
P_2	14/40	8/120	18
P_3	26	24	16/90

→ Minimum transportation Cost = 2,240 + 720 + 560 + 960 + 1440
= 5,920 //

Q-4 Solve the following unbalanced transportation problem for the minimum transportation cost:

	A	B	C	Supply
X	4	8	8	76
Y	16	24	16	82
Z	8	16	24	77
Demand	72	102	41	

	A	B	C	D	Supply
X	4	8	8	0	76
Y	16	24	16	0	82
Z	8	16	24	0	77
Demand	72	102	41	20	

→ Now $\Sigma a_i = \Sigma b_j = 235$
→ This is balanced transportation problem.

⇒ VAM:-

	A	B	C	D	Supply	Penalties			
X	4/	8/76	8/	0/	76/0	4	4	0	–
Y	16/	24/21	16/41	0/20	82/61/20	16	0	8	8
Z	8/72	16/5	24/	0/	77/5/0	8	8	8	8
Demand	72/0	102/26/0	41/0	20/0					
	4	8	8	0					
Penalties	4	8	8	–					
	–	8	8	–					
	–	8	8	–					

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→ Total Cost = 608 + 504 + 656 + 0 + 576 + 80 = 2,424//

→ So IBFS is

	A	B	C	D
X	4	8/76	8	0
Y	16	24/21	16/41	0/20
Z	8/72	16/5	24	0

→ Test for optimality :-

	A	B	C	D	
X	4/4	(8)	8/8	16/0	$U_1 = 0$
Y	0/16	(24)	(16)	(0)	$U_2 = -16$
Z	(8)	(16)	16/24	8/0	$U_3 = -8$
	$V_1 = 0$	$V_2 = -8$	$V_3 = 0$	$V_4 = 16$	

→ Since all $d_{ij} \geq 0$
So, final optimal sol^n is arrived.

	A	B	C	D
X	4	8/76	8	0
Y	16	24/21	16/41	0/20
Z	8/72	16/5	24	0

→ Minimum transportation Cost = 608 + 504 + 656 + 576 + 80
= 2424//

Q-5 Solve the following unbalanced transportation problem for the minimum transportation cost:

	D_1	D_2	D_3	D_4	Supply
P_1	25	25	24.50	25.50	2,75,000
P_2	24.75	25.50	26	26	5,50,000
P_3	24.25	26.75	25	24.50	6,60,000
Demand	1,10,000	2,20,000	3,30,000	4,40,000	

	D_1	D_2	D_3	D_4	D_5	Supply
P_1	25	25	24.50	25.50	0	2,75,000
P_2	24.75	25.50	26	26	0	5,50,000
P_3	24.25	26.75	25	24.50	0	6,60,000
Demand	1,10,000	2,20,000	3,30,000	4,40,000	3,85,000	

→ Here $\sum a_i = \sum b_j = 3,960,000$
This is balanced transportation problem.

⇒ VAM

	D_1	D_2	D_3	D_4	D_5	Supply						
P_1	25	25/55000	24.5/220000	25.5	0	275000/55000/0	24.5	0.5	0.5	0.5	0.5	25	-
P_2	24.75	25/165000	26	26	0/385000	550000/165000	24.75	0.75	0.75	0.5	0.5	25	-
P_3	24.25/110000	26.75	25/110000	24.5/440000	0	660000/220000/110000/0	24.25	0.5	0.75	1.75	-	-	
Demand	110000/0	220000/55000/0	330000/220000/0	440000/0	385000/0								
	0.5	0.5	0.5	1	0								
Penalties	0.5	0.5	0.5	1	-								
	0.5	0.5	0.5	-	-								
	-	0.5	0.5	-	-								
	-	0.5	1.5	-	-								
	-	0.5	-	-	-								

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→ Total Cost = 1375000 + 5390000 + 4207500 + 0 + 2667500 + 2750000 + 10780000 ₹
= 27170000 //

⇒ So IBFS is

	D_1	D_2	D_3	D_4	D_5
P_1	25	25/55000	24.5/220000	25.5	0
P_2	24.75	25.5/165000	26	26	0/385000
P_3	24.25/110000	26.75	25/110000	24.5/440000	0

→ Test for optimality

	D_1	D_2	D_3	D_4	D_5	
P_1	1.25/25	(25)	(24.5)	1.5/25.5	0.5/0	$U_1 = 0$
P_2	0.5/24.75	(25.5)	1/26	1.5/26	(0)	$U_2 = -0.5$
P_3	(24.25)	1.25/26.75	(25)	(24.5)	0/0	$U_3 = -0.5$
	$V_1 = -23.75$	$V_2 = -25$	$V_3 = -24.5$	$V_4 = -24$	$V_5 = 0.5$	

→ Since all $d_{ij} \geq 0$
So, final optimal soln is arrived

	D_1	D_2	D_3	D_4	D_5
P_1	25	25/55000	24.5/220000	25.5	0
P_2	24.75	25.5/165000	26	26	0/385000
P_3	24.25/110000	26.75	25/110000	24.5/440000	0

→ Minimum transportation Cost = 1375000 + 5390000 + 4207500 + 0 + 2667500 + 2750000 + 10780000
∴ Minimum transportation Cost = 27170000 //

Q-6 Solve the following transportation problem for the minimum transportation cost:

	D_1	D_2	D_3	D_4	Supply
P_1	8	9	6	3	18
P_2	6	11	5	10	20
P_3	3	8	7	9	18
Demand	15	16	12	13	

-> Here $\Sigma a_i = \Sigma b_j = 56$

-> This is balanced transportation problem.

-> VAM :-

	D_1	D_2	D_3	D_4	Supply	Penalties			
P_1	8/	(9/5)	6/	(3/13)	18/15/0	3	2	3	9
P_2	6/	(11/8)	(5/12)	10/	20/8/0	1	1	6	11
P_3	(3/15)	(8/3)	7/	9/	18/3/0	4	4	1	8
Demand	15/0	16/8/0	12/0	13/0					
	3	1	1	6					
Penalties	3	1	1	-					
	-	1	1	-					
	-	1	-	-					

-> Total Cost - 45 + 39 + 88 + 60 + 45 + 24 = 301 //

→ So IBFS is

	D_1	D_2	D_3	D_4
S_1	8	9/5	6	3/13
S_2	6	11/8	5/12	10
S_3	3/15	8/3	7	9

→ Test for optimality:-

	D_1	D_2	D_3	D_4	
S_1	4/8	(9)	3/6	(3)	$U_1 = 0$
S_2	0/6	(11)	(5)	5/10	$U_2 = -2$
S_3	(3)	(8)	5/7	7/9	$U_3 = 1$
	$V_1 = -4$	$V_2 = -9$	$V_3 = -3$	$V_4 = -3$	

→ Since all $d_{ij} \geq 0$
So final sol^n is arrived

	D_1	D_2	D_3	D_4
S_1	8	9/5	6	3/13
S_2	6	11/8	5/12	10
S_3	3/15	8/3	7	9

→ Minimum transportation Cost = 45 + 39 + 88 + 60 + 45
= 301 //

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Q-7 Solve the following unbalanced transportation for the minimum transportation cost:

	D_1	D_2	D_3	D_4	Supply
P_1	90	90	100	100	200
P_2	50	70	130	85	100
Demand	75	100	100	30	

	D_1	D_2	P_3	D_4	Supply
P_1	90	90	100	100	200
P_2	50	70	130	85	100
P_3	0	0	0	0	5
Demand	75	100	100	30	

→ Now $\Sigma a_i : \Sigma b_j = 305$
→ This is balanced transportation problem.

⇒ VAM

	D_1	D_2	D_3	D_4	Supply	Penalties				
P_1	90/	(90/75)	(100/95)	(100/30)	200/105/0	10	10	10	10	10
P_2	(50/75)	(70/25)	130/	85/	100/25/0	20	20	15	15	-
P_3	0/	0/	(0/5)	0/	5/0	5	-	-	-	-
Demand	75/0	100/75/0	100/95/0	30/0						
	50	70	100	25						
Penalties	40	20	30	15						
	-	20	30	15						
	-	20	-	15						
	-	90	-	100						

→ Total Cost = 6750 + 9500 + 3000 + 3750 + 1750
= 24,750 //

→ So IBFS is

	D_1	D_2	D_3	D_4	Supply
P_1	90	90/75	100/95	100/30	200
P_2	50/75	70/25	130	85	100
P_3	0	0	0/5	0	5
Demand	75	100	100	30	

→ Test for optimality :-

	D_1	D_2	D_3	D_4	Supply
P_1	20/90	(90)	(100)	(100)	$U_1 = 0$
P_2	(50)	(70)	50/130	5/85	$U_2 = 20$
P_3	20/0	10/0	(0)	0/0	$U_3 = 100$
Demand	$V_1 = -70$	$V_2 = -90$	$V_3 = -100$	$V_4 = -100$	

→ Since all $d_{ij} \geq 0$
So final optimal soln is arrived

	D_1	D_2	D_3	D_4
P_1	90	90/75	100/95	100/30
P_2	50/75	70/25	130	85
P_3	0	0	0/5	0

→ Minimum transportation Cost = 6750 + 9500 + 3000 + 3750 + 1750
= 24,750 //

Q-8 Solve the following transportation problem for the minimum transportation cost:

	D_1	D_2	D_3	D_4	Supply
S_1	6	1	9	3	70
S_2	11	5	2	8	55
S_3	10	12	4	7	90
Demand	85	35	50	45	

→ Here $\Sigma a_i = \Sigma b_j = 205$
→ This is balanced transportation problem.

⇒ VAM :-

	D_1	D_2	D_3	D_4	Supply	Penalties				
S_1	6/	(1/35)	9/	(3/35)	70/35/0	2	3	3	-	-
S_2	(11/5)	5/	(2/50)	8/	55/5/0	3	6	3	3	11
S_3	(10/80)	12/	4/	(7/10)	90/80/0	3	3	3	3	10
Demand	85/0	35/0	50/0	45/10/0						
	4	4	2	4						
Penalties	4	-	2	4						
	4	-	-	4						
	1	-	-	1						
	1	-	-	-						

→ Total Cost = 35 + 105 + 55 + 100 + 800 + 70 = 1165 //

→ So IBFS is

	D_1	D_2	D_3	D_4
S_1	6	1/35	9	3/35
S_2	11/5	5	2/50	8
S_3	10/80	12	4	7/10

→ Test for optimality :-

	D_1	D_2	D_3	D_4	
S_1	0/6	(1)	12/9	(3)	$U_1 = 0$
S_2	(11)	-1/5	(2)	0/8	$U_2 = 5$
S_3	(10)	7/12	3/4	(7)	$U_3 = -4$
	$V_1 = -6$	$V_2 = -1$	$V_3 = 3$	$V_4 = -3$	

→ Since we get one entry as negative so present soln is not optimal.

⇒ Improvisation of soln

	D_1	D_2	D_3	D_4
S_1		• (x-35)		• (x+35)
S_2	• (x-5)	*	•	
S_3	• (x+80)			• (x-10)

Let $x = \min\{\text{no. with } -x\} = \min\{35, 10, 5\} = 5$

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→ Test for optimality
→ Improvised soln is

	D_1	D_2	D_3	D_4
S_1	6	1/30	9	3/40
S_2	11	5/5	2/80	8
S_3	10/85	12	4	7/5

→ Test for optimality

	D_1	D_2	D_3	D_4	
S_1	0/6	(1)	11/9	(3)	$U_1 = 0$
S_2	1/11	(5)	(2)	1/8	$U_2 = -4$
S_3	(10)	7/12	2/4	(7)	$U_3 = -4$
	$V_1 = -6$	$V_2 = -1$	$V_3 = 2$	$V_4 = -3$	

→ Since all $d_{ij} \geq 0$ present soln is optimal.

→ Minimum transportation Cost = 30+120+25+100+850+35
= 1,160 //

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Q-9 Define an unbalanced transportation problem. Solve the following unbalanced transportation problem for the minimum transportation cost.

	A	B	C	D	Supply
W	20	21	16	18	10
X	17	28	14	16	9
Y	29	23	19	20	7
Demand	6	10	4	5	

	A	B	C	D	E	Supply
W	20	21	16	18	0	10
X	17	28	14	16	0	9
Y	29	23	19	20	0	7
Demand	6	10	4	5	1	

→ Here $\Sigma a_i = \Sigma b_j = 26$

→ This is balanced transportation problem.

VAM:

	A	B	C	D	E	Supply	Penalties			
W	20/	(24/4)	(16/1)	(18/5)	0/	10/9/4/0	16	2	2	2
X	(17/6)	28/	(14/3)	16/	0/	9/3/0	14	2	2	-
Y	29/	(23/6)	19/	20/	(0/1)	7/6/0	19	1	1	1
Demand	6/0	10/4/0	4/1/0	5/0	1/0					
Penalties	3	2	2	2	0					
	3	2	2	2	-					
	-	2	2	2	-					
	-	2	3	2	-					
	-	2	-	2	-					

→ Total Cost: 84+16+90+102+42+138 = 472//

→ So IBFS is

	A	B	C	D	E
W	20	21/4	16/1	18/5	0
X	17/6	28	14/3	16	0
Y	29	23/6	19	20	0/1

→ Test for optimality

	A	B	C	D	E	
W	(2) 20	(21)	(16)	(18)	(2) 0	$U_1 = 0$
X	(17)	(9) 28	(14)	(0) 16	(4) 0	$U_2 = 2$
Y	(8) 0	(23)	(1) 19	(0) 20	(0)	$U_3 = -2$
	$V_1 = -19$	$V_2 = -21$	$V_3 = -16$	$V_4 = -18$	$V_5 = 2$	

→ Since all $d_{ij} \geq 0$, so present sol^n is optimal
→ Minimum transportation Cost = 84+16+90+102+42+138
= 472//

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Q.10 Solve the following transportation problem for the minimum transportation cost:

	D_1	D_2	D_3	D_4	Supply
P_1	8	9	6	3	36
P_2	6	11	5	10	40
P_3	3	8	7	9	36
Demand	30	32	24	26	

→ Here $\Sigma a_i = \Sigma b_j = 112$

→ This is balanced transportation problem.

⇒ VAM:-

	D_1	D_2	D_3	D_4	Supply	Penalties			
P_1	8/	9/10	6/	3/26	36/10/0	3	2	3	9
P_2	6/	11/16	5/24	10/	40/16/0	1	1	6	4
P_3	3/30	8/6	7/	9/	36/6/0	4	4	1	8
Demand	30/0	32/16/0	24/0	26/0					
Penalties	3	1	1	6					
	3	1	1	–					
	–	1	1	–					
	–	1	–	–					

→ Total Cost: 90+78+176+120+90+48 = 602//

→ So IBFS is

	D_1	D_2	D_3	D_4
P_1	8	9/10	6	3/26
P_2	6	11/16	5/24	10
P_3	3/30	8/6	7	9

→ Test for optimality:-

	D_1	D_2	D_3	D_4	
P_1	4/8	(9)	3/6	(3)	$U_1 = 0$
P_2	0/6	(11)	(5)	5/10	$U_2 = -2$
P_3	(3)	(6)	5/7	7/9	$U_3 = 1$
	$V_1 = -4$	$V_2 = -9$	$V_3 = -3$	$V_4 = -3$	

→ Since all $d_{ij} \geq 0$
So final soln is arrived.

	D_1	D_2	D_3	D_4
P_1	8	9/10	6	3/26
P_2	6	11/16	5/24	10
P_3	3/30	8/6	7	9

→ Minimum transportation Cost = 90+78+176+120+90+48 = 602 //