Print This Solution Close This Solution

Find solution using dual-simplex method
MAX Z = -2x1 - 2x2 - 4x3
subject to
2x1 + 3x2 + 5x3 >= 2
3x1 + x2 + 7x3 <= 3
x1 + 4x2 + 6x3 <= 5
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = -2x_1 - 2x_2 - 4x_3$$

subject to

$$2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$

and $x_1, x_2, x_3 \geq 0$;

In order to apply the dual simplex method, convert all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -2x_1 - 2x_2 - 4x_3$$

subject to

$$-2x_1 - 3x_2 - 5x_3 \leq -2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = -2x_1 - 2x_2 - 4x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$-2x_1 - 3x_2 - 5x_3 + S_1 = -2$$
$$3x_1 + x_2 + 7x_3 + S_2 = 3$$
$$x_1 + 4x_2 + 6x_3 + S_3 = 5$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iteration-1		C_j	-2	-2	-4	0	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**
S_1	0	-2	-2	**(-3)**	-5	1	0	0
S_2	0	3	3	1	7	0	1	0
S_3	0	5	1	4	6	0	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-2	-2	-4	0	0	0
		Ratio $= \frac{C_j - Z_j}{S_1,j}$ and $S_1,j < 0$	1	$\frac{2}{3}$ ↑	$\frac{4}{5}$	---	---	---

Minimum negative X_B is -2 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is $\frac{2}{3}$ and its column index is 2. So, the entering variable is x_2.

∴ The pivot element is -3.

Entering $= x_2$, Departing $= S_1$, Key Element $= -3$

$R_1(\text{new}) = R_1(\text{old}) \div -3$

$R_2(\text{new}) = R_2(\text{old}) - R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - 4R_1(\text{new})$

Iteration-2		C_j	-2	-2	-4	0	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**
x_2	-2	$\frac{2}{3}$	$\frac{2}{3}$	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0
S_2	0	$\frac{7}{3}$	$\frac{7}{3}$	0	$\frac{16}{3}$	$\frac{1}{3}$	1	0
S_3	0	$\frac{7}{3}$	$-\frac{5}{3}$	0	$-\frac{2}{3}$	$\frac{4}{3}$	0	1
$Z = -\frac{4}{3}$		Z_j	$-\frac{4}{3}$	**-2**	$-\frac{10}{3}$	$\frac{2}{3}$	**0**	**0**

		$C_j - Z_j$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
		Ratio	---	---	---	---	---	---

Since all $C_j - Z_j \le 0$ and all $X_{Bi} \ge 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :

$x_1 = 0, x_2 = \frac{2}{3}, x_3 = 0$

Max $Z = -\frac{4}{3}$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MAX Z = -x1 - 2x2
subject to
-2x1 - x2 <= -4
-x1 - 2x2 <= -7
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = -x_1 - 2x_2$$

subject to

$$-2x_1 - x_2 \le -4$$
$$-x_1 - 2x_2 \le -7$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -x_1 - 2x_2 + 0S_1 + 0S_2$$

subject to

$$-2x_1 - x_2 + S_1 = -4$$
$$-x_1 - 2x_2 + S_2 = -7$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	-1	-2	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-4	-2	-1	1	0
S_2	0	-7	**(-1)**	-2	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-1	-2	0	0
		Ratio $= \frac{C_j - Z_j}{S_{2,j}}$ and $S_{2,j} < 0$	1 ↑	1	---	---

Minimum negative X_B is -7 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is 1 and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is -1.

Entering $= x_1$, Departing $= S_2$, Key Element $= -1$

$R_2(\text{new}) = R_2(\text{old}) \div -1$

$R_1(\text{new}) = R_1(\text{old}) + 2R_2(\text{new})$

Iteration-2		C_j	-1	-2	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**
S_1	0	10	0	3	1	-2
x_1	-1	7	1	2	0	-1
$Z = -7$		Z_j	**-1**	**-2**	**0**	**1**
		$C_j - Z_j$	0	0	0	-1
		Ratio	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 7, x_2 = 0$

Max $Z = -7$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 2x1 + x2
subject to
3x2 >= 6
3x1 + 2x2 >= 8
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 2x_1 + x_2$$

subject to

$$3x_2 \geq 6$$
$$3x_1 + 2x_2 \geq 8$$

and $x_1, x_2 \geq 0;$

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -2x_1 - x_2$$

subject to

$$-3x_2 \leq -6$$
$$-3x_1 - 2x_2 \leq -8$$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -2x_1 - x_2 + 0S_1 + 0S_2$$

subject to

$$-3x_2 + S_1 = -6$$
$$-3x_1 - 2x_2 + S_2 = -8$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	-2	-1	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-6	0	-3	1	0

S_2	0	-8	-3	**(-2)**	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-2	-1	0	0
		Ratio $= \frac{C_j - Z_j}{S_2,j}$ and $S_2,j < 0$	$\frac{2}{3}$	$\frac{1}{2}$ ↑	---	---

Minimum negative X_B is -8 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is $\frac{1}{2}$ and its column index is 2. So, the entering variable is x_2.

$\therefore$ The pivot element is -2.

Entering $= x_2$, Departing $= S_2$, Key Element $= -2$

$R_2(\text{new}) = R_2(\text{old}) \div -2$

$R_1(\text{new}) = R_1(\text{old}) + 3R_2(\text{new})$

Iteration-2		C_j	-2	-1	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	6	$\frac{9}{2}$	0	1	$-\frac{3}{2}$
x_2	-1	4	$\frac{3}{2}$	1	0	$-\frac{1}{2}$
$Z = -4$		Z_j	$-\frac{3}{2}$	**-1**	**0**	$\frac{1}{2}$
		$C_j - Z_j$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$
		Ratio	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 4$

Max $Z = -4$

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 3x1 + 4x2
subject to
2x1 + 3x2 >= 90
4x1 + 3x2 >= 120
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 3x_1 + 4x_2$$

subject to

$$2x_1 + 3x_2 \geq 90$$

$$4x_1 + 3x_2 \geq 120$$

and $x_1, x_2 \geq 0;$

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -3x_1 - 4x_2$$

subject to

$$-2x_1 - 3x_2 \leq -90$$

$$-4x_1 - 3x_2 \leq -120$$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type '$\leq$' we should add slack variable S_1

2. As the constraint 2 is of type '$\leq$' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -3x_1 - 4x_2 + 0S_1 + 0S_2$$

subject to

$$-2x_1 - 3x_2 + S_1 = -90$$

$$-4x_1 - 3x_2 + S_2 = -120$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	-3	-4	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**
S_1	0	-90	-2	-3	1	0

S_2	0	-120	**(-4)**	-3	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-3	-4	0	0
		Ratio $= \frac{C_j - Z_j}{S_2, j}$ and $S_2, j < 0$	$\frac{3}{4}$ ↑	$\frac{4}{3}$	---	---

Minimum negative X_B is -120 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is $\frac{3}{4}$ and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is -4.

Entering $= x_1$, Departing $= S_2$, Key Element $= -4$

$R_2(\text{new}) = R_2(\text{old}) \div -4$

$R_1(\text{new}) = R_1(\text{old}) + 2R_2(\text{new})$

Iteration-2		C_j	-3	-4	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**
S_1	0	-30	0	**$\left(-\frac{3}{2}\right)$**	1	$-\frac{1}{2}$
x_1	-3	30	1	$\frac{3}{4}$	0	$-\frac{1}{4}$
$Z = -90$		Z_j	**-3**	**$-\frac{9}{4}$**	**0**	**$\frac{3}{4}$**
		$C_j - Z_j$	0	$-\frac{7}{4}$	0	$-\frac{3}{4}$
		Ratio $= \frac{C_j - Z_j}{S_1, j}$ and $S_1, j < 0$	---	$\frac{7}{6}$ ↑	---	$\frac{3}{2}$

Minimum negative X_B is -30 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is $\frac{7}{6}$ and its column index is 2. So, the entering variable is x_2.

∴ The pivot element is $-\frac{3}{2}$.

Entering $= x_2$, Departing $= S_1$, Key Element $= -\frac{3}{2}$

$R_1(\text{new}) = R_1(\text{old}) \times -\frac{2}{3}$

$R_2(\text{new}) = R_2(\text{old}) - \frac{3}{4} R_1(\text{new})$

Iteration-3		C_j	-3	-4	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
x_2	-4	20	0	1	$-\frac{2}{3}$	$\frac{1}{3}$
x_1	-3	15	1	0	$\frac{1}{2}$	$-\frac{1}{2}$
$Z = -125$		Z_j	**-3**	**-4**	$\frac{7}{6}$	$\frac{1}{6}$
		$C_j - Z_j$	0	0	$-\frac{7}{6}$	$-\frac{1}{6}$
		Ratio	---	---	---	---

Since all $C_j - Z_j \le 0$ and all $X_{Bi} \ge 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 15, x_2 = 20$

Max $Z = -125$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 2x1 + 2x2 + 4x3
subject to
2x1 + 3x2 + 5x3 >= 2
3x1 + x2 + 7x3 <= 3
x1 + 4x2 + 6x3 <= 5
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Min } Z = 2x_1 + 2x_2 + 4x_3$$

subject to

$$2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$

and $x_1, x_2, x_3 \geq 0$;

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -2x_1 - 2x_2 - 4x_3$$

subject to

$$-2x_1 - 3x_2 - 5x_3 \leq -2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = -2x_1 - 2x_2 - 4x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$-2x_1 - 3x_2 - 5x_3 + S_1 = -2$$
$$3x_1 + x_2 + 7x_3 + S_2 = 3$$
$$x_1 + 4x_2 + 6x_3 + S_3 = 5$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iteration-1		C_j	-2	-2	-4	0	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**
S_1	0	-2	-2	**(-3)**	-5	1	0	0
S_2	0	3	3	1	7	0	1	0
S_3	0	5	1	4	6	0	0	1
$Z = 0$		**Z_j**	**0**	**0**	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-2	-2	-4	0	0	0
		Ratio $= \frac{C_j - Z_j}{S_{1,j}}$ and $S_{1,j} < 0$	1	$\frac{2}{3}$ ↑	$\frac{4}{5}$	---	---	---

Minimum negative X_B is -2 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is $\frac{2}{3}$ and its column index is 2. So, the entering variable is x_2.

∴ The pivot element is -3.

Entering $= x_2$, Departing $= S_1$, Key Element $= -3$

$R_1(\text{new}) = R_1(\text{old}) \div -3$

$R_2(\text{new}) = R_2(\text{old}) - R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - 4R_1(\text{new})$

Iteration-2		C_j	-2	-2	-4	0	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**
x_2	-2	$\frac{2}{3}$	$\frac{2}{3}$	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0
S_2	0	$\frac{7}{3}$	$\frac{7}{3}$	0	$\frac{16}{3}$	$\frac{1}{3}$	1	0
S_3	0	$\frac{7}{3}$	$-\frac{5}{3}$	0	$-\frac{2}{3}$	$\frac{4}{3}$	0	1
$Z = -\frac{4}{3}$		**Z_j**	**$-\frac{4}{3}$**	**-2**	**$-\frac{10}{3}$**	**$\frac{2}{3}$**	**0**	**0**

		$C_j - Z_j$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
		Ratio	---	---	---	---	---	---

Since all $C_j - Z_j \le 0$ and all $X_{Bi} \ge 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :

$x_1 = 0, x_2 = \frac{2}{3}, x_3 = 0$

Max $Z = -\frac{4}{3}$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 2x1 + x2 + 4x3
subject to
2x1 + 3x2 + 3x3 >= 12
3x1 + 2x2 + x3 >= 18
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Min } Z = 2x_1 + x_2 + 4x_3$$

subject to

$$2x_1 + 3x_2 + 3x_3 \geq 12$$
$$3x_1 + 2x_2 + x_3 \geq 18$$

and $x_1, x_2, x_3 \geq 0;$

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -2x_1 - x_2 - 4x_3$$

subject to

$$-2x_1 - 3x_2 - 3x_3 \leq -12$$
$$-3x_1 - 2x_2 - x_3 \leq -18$$

and $x_1, x_2, x_3 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -2x_1 - x_2 - 4x_3 + 0S_1 + 0S_2$$

subject to

$$-2x_1 - 3x_2 - 3x_3 + S_1 = -12$$
$$-3x_1 - 2x_2 - x_3 + S_2 = -18$$

and $x_1, x_2, x_3, S_1, S_2 \geq 0$

Iteration-1		C_j	-2	-1	-4	0	0
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2
S_1	0	-12	-2	-3	-3	1	0

S_2	0	-18	-3	**(-2)**	-1	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-2	-1	-4	0	0
		Ratio $= \dfrac{C_j - Z_j}{S_{2},j}$ and $S_{2},j < 0$	$\frac{2}{3}$	$\frac{1}{2}$ ↑	4	---	---

Minimum negative X_B is -18 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is $\frac{1}{2}$ and its column index is 2. So, the entering variable is x_2.

∴ The pivot element is -2.

Entering $= x_2$, Departing $= S_2$, Key Element $= -2$

$R_2(\text{new}) = R_2(\text{old}) \div -2$

$R_1(\text{new}) = R_1(\text{old}) + 3R_2(\text{new})$

Iteration-2		C_j	-2	-1	-4	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**
S_1	0	15	$\frac{5}{2}$	0	$-\frac{3}{2}$	1	$-\frac{3}{2}$
x_2	-1	9	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$
$Z = -9$		Z_j	**$-\frac{3}{2}$**	**-1**	**$-\frac{1}{2}$**	**0**	**$\frac{1}{2}$**
		$C_j - Z_j$	$-\frac{1}{2}$	0	$-\frac{7}{2}$	0	$-\frac{1}{2}$
		Ratio	---	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 9, x_3 = 0$

Max $Z = -9$

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 3x1 + 2x2 + x3
subject to
2x1 + x2 + 4x3 >= 15
x1 + 4x2 + 3x3 >= 21
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Min } Z = 3x_1 + 2x_2 + x_3$$

subject to

$$2x_1 + x_2 + 4x_3 \geq 15$$
$$x_1 + 4x_2 + 3x_3 \geq 21$$

and $x_1, x_2, x_3 \geq 0$;

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -3x_1 - 2x_2 - x_3$$

subject to

$$-2x_1 - x_2 - 4x_3 \leq -15$$
$$-x_1 - 4x_2 - 3x_3 \leq -21$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type '$\leq$' we should add slack variable S_1

2. As the constraint 2 is of type '$\leq$' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -3x_1 - 2x_2 - x_3 + 0S_1 + 0S_2$$

subject to

$$-2x_1 - x_2 - 4x_3 + S_1 = -15$$
$$-x_1 - 4x_2 - 3x_3 + S_2 = -21$$

and $x_1, x_2, x_3, S_1, S_2 \geq 0$

Iteration-1		C_j	-3	-2	-1	0	0
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2
S_1	0	-15	-2	-1	-4	1	0

S_2	0	-21	-1	-4	**(-3)**	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-3	-2	-1	0	0
		Ratio $= \dfrac{C_j - Z_j}{S_{2},j}$ and $S_{2},j < 0$	3	$\frac{1}{2}$	$\frac{1}{3}$ ↑	---	---

Minimum negative X_B is -21 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is $\frac{1}{3}$ and its column index is 3. So, the entering variable is x_3.

∴ The pivot element is -3.

Entering $= x_3$, Departing $= S_2$, Key Element $= -3$

$R_2(\text{new}) = R_2(\text{old}) \div -3$

$R_1(\text{new}) = R_1(\text{old}) + 4R_2(\text{new})$

Iteration-2		C_j	-3	-2	-1	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**
S_1	0	13	$-\frac{2}{3}$	$\frac{13}{3}$	0	1	$-\frac{4}{3}$
x_3	-1	7	$\frac{1}{3}$	$\frac{4}{3}$	1	0	$-\frac{1}{3}$
$Z = -7$		Z_j	**$-\frac{1}{3}$**	**$-\frac{4}{3}$**	**-1**	**0**	**$\frac{1}{3}$**
		$C_j - Z_j$	$-\frac{8}{3}$	$-\frac{2}{3}$	0	0	$-\frac{1}{3}$
		Ratio	---	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 0, x_3 = 7$

Max $Z = -7$

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 3x1 + x2 + 2x3
subject to
4x1 + x2 + 4x3 >= 12
x1 + 3x2 + 4x3 >= 10
2x1 + 2x2 + x3 >= 6
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Min } Z = 3x_1 + x_2 + 2x_3$$

subject to

$$4x_1 + x_2 + 4x_3 \geq 12$$
$$x_1 + 3x_2 + 4x_3 \geq 10$$
$$2x_1 + 2x_2 + x_3 \geq 6$$

and $x_1, x_2, x_3 \geq 0$;

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -3x_1 - x_2 - 2x_3$$

subject to

$$-4x_1 - x_2 - 4x_3 \leq -12$$
$$-x_1 - 3x_2 - 4x_3 \leq -10$$
$$-2x_1 - 2x_2 - x_3 \leq -6$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = -3x_1 - x_2 - 2x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$-4x_1 - x_2 - 4x_3 + S_1 = -12$$
$$-x_1 - 3x_2 - 4x_3 + S_2 = -10$$
$$-2x_1 - 2x_2 - x_3 + S_3 = -6$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iteration-1		C_j	-3	-1	-2	0	0	0
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
S_1	0	-12	-4	-1	(-4)	1	0	0
S_2	0	-10	-1	-3	-4	0	1	0
S_3	0	-6	-2	-2	-1	0	0	1
$Z = 0$		Z_j	0	0	0	0	0	0
		$C_j - Z_j$	-3	-1	-2	0	0	0
		Ratio $= \frac{C_j - Z_j}{S_1, j}$ and $S_1, j < 0$	$\frac{3}{4}$	1	$\frac{1}{2}$ ↑	---	---	---

Minimum negative X_B is -12 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is $\frac{1}{2}$ and its column index is 3. So, the entering variable is x_3.

∴ The pivot element is -4.

Entering $= x_3$, Departing $= S_1$, Key Element $= -4$

$R_1(\text{new}) = R_1(\text{old}) \div -4$

$R_2(\text{new}) = R_2(\text{old}) + 4R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + R_1(\text{new})$

Iteration-2		C_j	-3	-1	-2	0	0	0
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
x_3	-2	3	1	$\frac{1}{4}$	1	$-\frac{1}{4}$	0	0
S_2	0	2	3	-2	0	-1	1	0
S_3	0	-3	-1	$\left(-\frac{7}{4}\right)$	0	$-\frac{1}{4}$	0	1
$Z = -6$		Z_j	-2	$-\frac{1}{2}$	-2	$\frac{1}{2}$	0	0

		$C_j - Z_j$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0
		Ratio $= \frac{C_j - Z_j}{S_3, j}$ and $S_3, j < 0$	1	$\frac{2}{7}$ ↑	---	2	---	---

Minimum negative X_B is -3 and its row index is 3. So, the leaving basis variable is S_3.

Minimum positive ratio is $\frac{2}{7}$ and its column index is 2. So, the entering variable is x_2.

∴ The pivot element is $-\frac{7}{4}$.

Entering $= x_2$, Departing $= S_3$, Key Element $= -\frac{7}{4}$

$R_3(\text{new}) = R_3(\text{old}) \times -\frac{4}{7}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{4} R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) + 2R_3(\text{new})$

Iteration-3		C_j	-3	-1	-2	0	0	0
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
x_3	-2	$\frac{18}{7}$	$\frac{6}{7}$	0	1	$-\frac{2}{7}$	0	$\frac{1}{7}$
S_2	0	$\frac{38}{7}$	$\frac{29}{7}$	0	0	$-\frac{5}{7}$	1	$-\frac{8}{7}$
x_2	-1	$\frac{12}{7}$	$\frac{4}{7}$	1	0	$\frac{1}{7}$	0	$-\frac{4}{7}$
$Z = -\frac{48}{7}$		Z_j	$-\frac{16}{7}$	-1	-2	$\frac{3}{7}$	0	$\frac{2}{7}$
		$C_j - Z_j$	$-\frac{5}{7}$	0	0	$-\frac{3}{7}$	0	$-\frac{2}{7}$
		Ratio	---	---	---	---	---	---

Since all $C_j - Z_j \le 0$ and all $X_{Bi} \ge 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :

$x_1 = 0, x_2 = \frac{12}{7}, x_3 = \frac{18}{7}$

Max $Z = -\frac{48}{7}$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 4x1 + 2x2
subject to
4x1 + x2 >= 14
x1 + 3x2 >= 12
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 4x_1 + 2x_2$$

subject to

$$4x_1 + x_2 \geq 14$$
$$x_1 + 3x_2 \geq 12$$

and $x_1, x_2 \geq 0;$

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -4x_1 - 2x_2$$

subject to

$$-4x_1 - x_2 \leq -14$$
$$-x_1 - 3x_2 \leq -12$$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -4x_1 - 2x_2 + 0S_1 + 0S_2$$

subject to

$$-4x_1 - x_2 + S_1 = -14$$
$$-x_1 - 3x_2 + S_2 = -12$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	-4	-2	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-14	(-4)	-1	1	0

S_2	0	-12	-1	-3	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-4	-2	0	0
		Ratio $= \frac{C_j - Z_j}{S_1,j}$ and $S_1,j < 0$	1 ↑	2	---	---

Minimum negative X_B is -14 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is 1 and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is -4.

Entering $= x_1$, Departing $= S_1$, Key Element $= -4$

$R_1(\text{new}) = R_1(\text{old}) \div -4$

$R_2(\text{new}) = R_2(\text{old}) + R_1(\text{new})$

Iteration-2		C_j	-4	-2	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
x_1	-4	$\frac{7}{2}$	1	$\frac{1}{4}$	$-\frac{1}{4}$	0
S_2	0	$-\frac{17}{2}$	0	$\left(-\frac{11}{4}\right)$	$-\frac{1}{4}$	1
$Z = -14$		Z_j	**-4**	**-1**	**1**	**0**
		$C_j - Z_j$	0	-1	-1	0
		Ratio $= \frac{C_j - Z_j}{S_2,j}$ and $S_2,j < 0$	---	$\frac{4}{11}$ ↑	4	---

Minimum negative X_B is $-\frac{17}{2}$ and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is $\frac{4}{11}$ and its column index is 2. So, the entering variable is x_2.

$\therefore$ The pivot element is $-\frac{11}{4}$.

Entering $= x_2$, Departing $= S_2$, Key Element $= -\frac{11}{4}$

$R_2(\text{new}) = R_2(\text{old}) \times -\frac{4}{11}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{4} R_2(\text{new})$

Iteration-3		C_j	-4	-2	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**
x_1	-4	$\frac{30}{11}$	1	0	$-\frac{3}{11}$	$\frac{1}{11}$
x_2	-2	$\frac{34}{11}$	0	1	$\frac{1}{11}$	$-\frac{4}{11}$
$Z = -\frac{188}{11}$		**Z_j**	**-4**	**-2**	**$\frac{10}{11}$**	**$\frac{4}{11}$**
		$C_j - Z_j$	0	0	$-\frac{10}{11}$	$-\frac{4}{11}$
		Ratio	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :

$x_1 = \frac{30}{11}, x_2 = \frac{34}{11}$

Max $Z = -\frac{188}{11}$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 5x1 + 8x2
subject to
2x1 + 3x2 >= 4
x1 - 2x2 >= 5
x1 + x2 >= 12
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 5x_1 + 8x_2$$

subject to

$$2x_1 + 3x_2 \geq 4$$
$$x_1 - 2x_2 \geq 5$$
$$x_1 + x_2 \geq 12$$

and $x_1, x_2 \geq 0$;

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -5x_1 - 8x_2$$

subject to

$$-2x_1 - 3x_2 \leq -4$$
$$-x_1 + 2x_2 \leq -5$$
$$-x_1 - x_2 \leq -12$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = -5x_1 - 8x_2 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$-2x_1 - 3x_2 + S_1 = -4$$
$$-x_1 + 2x_2 + S_2 = -5$$
$$-x_1 - x_2 + S_3 = -12$$

and $x_1, x_2, S_1, S_2, S_3 \geq 0$

Iteration-1		C_j	-5	-8	0	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3
S_1	0	-4	-2	-3	1	0	0
S_2	0	-5	-1	2	0	1	0
S_3	0	-12	**(-1)**	-1	0	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-5	-8	0	0	0
		Ratio $= \frac{C_j - Z_j}{S_{3,j}}$ and $S_{3,j} < 0$	5 ↑	8	---	---	---

Minimum negative X_B is -12 and its row index is 3. So, the leaving basis variable is S_3.

Minimum positive ratio is 5 and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is -1.

Entering $= x_1$, Departing $= S_3$, Key Element $= -1$

$R_3(\text{new}) = R_3(\text{old}) \div -1$

$R_1(\text{new}) = R_1(\text{old}) + 2R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) + R_3(\text{new})$

Iteration-2		C_j	-5	-8	0	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3
S_1	0	20	0	-1	1	0	-2
S_2	0	7	0	3	0	1	-1
x_1	-5	12	1	1	0	0	-1
$Z = -60$		Z_j	**-5**	**-5**	**0**	**0**	**5**
		$C_j - Z_j$	0	-3	0	0	-5
		Ratio	---	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 12, x_2 = 0$

Max $Z = -60$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 10x1 + 20x2
subject to
x1 + 2x2 >= 6
x1 + 4x2 >= 8
and x1,x2 >= 0

Solution:
Problem is

$\text{Min } Z = 10x_1 + 20x_2$

subject to

$x_1 + 2x_2 \geq 6$

$x_1 + 4x_2 \geq 8$

and $x_1, x_2 \geq 0;$

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$\text{Max } Z = -10x_1 - 20x_2$

subject to

$-x_1 - 2x_2 \leq -6$

$-x_1 - 4x_2 \leq -8$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type '$\leq$' we should add slack variable S_1

2. As the constraint 2 is of type '$\leq$' we should add slack variable S_2

After introducing slack variables

$\text{Max } Z = -10x_1 - 20x_2 + 0S_1 + 0S_2$

subject to

$-x_1 - 2x_2 + S_1 = -6$

$-x_1 - 4x_2 + S_2 = -8$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	-10	-20	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-6	-1	-2	1	0

S_2	0	-8	-1	(-4)	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-10	-20	0	0
		Ratio $= \frac{C_j - Z_j}{S_2, j}$ and $S_2, j < 0$	10	5 ↑	---	---

Minimum negative X_B is -8 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is 5 and its column index is 2. So, the entering variable is x_2.

∴ The pivot element is -4.

Entering $= x_2$, Departing $= S_2$, Key Element $= -4$

$R_2(\text{new}) = R_2(\text{old}) \div -4$

$R_1(\text{new}) = R_1(\text{old}) + 2R_2(\text{new})$

Iteration-2		C_j	-10	-20	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-2	$\left(-\frac{1}{2}\right)$	0	1	$-\frac{1}{2}$
x_2	-20	2	$\frac{1}{4}$	1	0	$-\frac{1}{4}$
$Z = -40$		Z_j	**-5**	**-20**	**0**	**5**
		$C_j - Z_j$	-5	0	0	-5
		Ratio $= \frac{C_j - Z_j}{S_1, j}$ and $S_1, j < 0$	10 ↑	---	---	10

Minimum negative X_B is -2 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is 10 and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is $-\frac{1}{2}$.

Entering $= x_1$, Departing $= S_1$, Key Element $= -\frac{1}{2}$

$R_1(\text{new}) = R_1(\text{old}) \times -2$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{4} R_1(\text{new})$

Iteration-3		C_j	-10	-20	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**
x_1	-10	4	1	0	-2	1
x_2	-20	1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$
$Z = -60$		**Z_j**	**-10**	**-20**	**10**	**0**
		$C_j - Z_j$	0	0	-10	0
		Ratio	---	---	---	---

Since all $C_j - Z_j \le 0$ and all $X_{Bi} \ge 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 4, x_2 = 1$

Max $Z = -60$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = 12x1 + 8x2
subject to
2x1 + 2x2 >= 6
3x1 + x2 >= 7
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 12x_1 + 8x_2$$

subject to

$$2x_1 + 2x_2 \geq 6$$
$$3x_1 + x_2 \geq 7$$

and $x_1, x_2 \geq 0;$

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -12x_1 - 8x_2$$

subject to

$$-2x_1 - 2x_2 \leq -6$$
$$-3x_1 - x_2 \leq -7$$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -12x_1 - 8x_2 + 0S_1 + 0S_2$$

subject to

$$-2x_1 - 2x_2 + S_1 = -6$$
$$-3x_1 - x_2 + S_2 = -7$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	-12	-8	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-6	-2	-2	1	0

S_2	0	-7	**(-3)**	-1	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-12	-8	0	0
		Ratio $= \frac{C_j - Z_j}{S_2, j}$ and $S_2, j < 0$	4 ↑	8	---	---

Minimum negative X_B is -7 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is 4 and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is -3.

Entering $= x_1$, Departing $= S_2$, Key Element $= -3$

$R_2(\text{new}) = R_2(\text{old}) \div -3$

$R_1(\text{new}) = R_1(\text{old}) + 2R_2(\text{new})$

Iteration-2		C_j	-12	-8	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**
S_1	0	$-\frac{4}{3}$	0	$\left(-\frac{4}{3}\right)$	1	$-\frac{2}{3}$
x_1	-12	$\frac{7}{3}$	1	$\frac{1}{3}$	0	$-\frac{1}{3}$
$Z = -28$		Z_j	**-12**	**-4**	**0**	**4**
		$C_j - Z_j$	0	-4	0	-4
		Ratio $= \frac{C_j - Z_j}{S_1, j}$ and $S_1, j < 0$	---	3 ↑	---	6

Minimum negative X_B is $-\frac{4}{3}$ and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is 3 and its column index is 2. So, the entering variable is x_2.

∴ The pivot element is $-\frac{4}{3}$.

Entering $= x_2$, Departing $= S_1$, Key Element $= -\frac{4}{3}$

$R_1(\text{new}) = R_1(\text{old}) \times -\frac{3}{4}$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{3} R_1(\text{new})$

Iteration-3		C_j	-12	-8	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**
x_2	-8	1	0	1	$-\frac{3}{4}$	$\frac{1}{2}$
x_1	-12	2	1	0	$\frac{1}{4}$	$-\frac{1}{2}$
$Z = -32$		**Z_j**	**-12**	**-8**	**3**	**2**
		$C_j - Z_j$	0	0	-3	-2
		Ratio	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 2, x_2 = 1$

Max $Z = -32$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = x1 + 2x2
subject to
2x1 + x2 >= 4
x1 + 2x2 <= 7
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = x_1 + 2x_2$$

subject to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 2x_2 \leq 7$$

and $x_1, x_2 \geq 0;$

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -x_1 - 2x_2$$

subject to

$$-2x_1 - x_2 \leq -4$$
$$x_1 + 2x_2 \leq 7$$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -x_1 - 2x_2 + 0S_1 + 0S_2$$

subject to

$$-2x_1 - x_2 + S_1 = -4$$
$$x_1 + 2x_2 + S_2 = 7$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	-1	-2	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-4	(-2)	-1	1	0

S_2	0	7	1	2	0	1
$\mathbf{Z = 0}$		$\mathbf{Z_j}$	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-1	-2	0	0
		Ratio $= \frac{C_j - Z_j}{S_1, j}$ and $S_1, j < 0$	$\frac{1}{2}$ ↑	2	---	---

Minimum negative X_B is -4 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is $\frac{1}{2}$ and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is -2.

Entering $= x_1$, Departing $= S_1$, Key Element $= -2$

$R_1(\text{new}) = R_1(\text{old}) \div -2$

$R_2(\text{new}) = R_2(\text{old}) - R_1(\text{new})$

Iteration-2		C_j	-1	-2	0	0
B	$\mathbf{C_B}$	$\mathbf{X_B}$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{S_1}$	$\mathbf{S_2}$
x_1	-1	2	1	$\frac{1}{2}$	$-\frac{1}{2}$	0
S_2	0	5	0	$\frac{3}{2}$	$\frac{1}{2}$	1
$\mathbf{Z = -2}$		$\mathbf{Z_j}$	**-1**	$\mathbf{-\frac{1}{2}}$	$\mathbf{\frac{1}{2}}$	**0**
		$C_j - Z_j$	0	$-\frac{3}{2}$	$-\frac{1}{2}$	0
		Ratio	---	---	---	---

Since all $C_j - Z_j \le 0$ and all $X_{Bi} \ge 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 2, x_2 = 0$

Max $Z = -2$

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = x1 + 2x2 + 2x3
subject to
x1 + x2 + 2x3 >= 12
x1 + 2x2 + 4x3 >= 14
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Min } Z = x_1 + 2x_2 + 2x_3$$

subject to

$$x_1 + x_2 + 2x_3 \geq 12$$
$$x_1 + 2x_2 + 4x_3 \geq 14$$

and $x_1, x_2, x_3 \geq 0$;

In order to apply the dual simplex method, convert Min Z to Max Z and all $\geq$ constraint to $\leq$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -x_1 - 2x_2 - 2x_3$$

subject to

$$-x_1 - x_2 - 2x_3 \leq -12$$
$$-x_1 - 2x_2 - 4x_3 \leq -14$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -x_1 - 2x_2 - 2x_3 + 0S_1 + 0S_2$$

subject to

$$-x_1 - x_2 - 2x_3 + S_1 = -12$$
$$-x_1 - 2x_2 - 4x_3 + S_2 = -14$$

and $x_1, x_2, x_3, S_1, S_2 \geq 0$

Iteration-1		C_j	-1	-2	-2	0	0
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2
S_1	0	-12	-1	-1	-2	1	0

S_2	0	-14	-1	-2	(-4)	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-1	-2	-2	0	0
		Ratio $= \frac{C_j - Z_j}{S_2,j}$ and $S_2,j < 0$	1	1	$\frac{1}{2}$ ↑	---	---

Minimum negative X_B is -14 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is $\frac{1}{2}$ and its column index is 3. So, the entering variable is x_3.

∴ The pivot element is -4.

Entering $= x_3$, Departing $= S_2$, Key Element $= -4$

$R_2(\text{new}) = R_2(\text{old}) \div -4$

$R_1(\text{new}) = R_1(\text{old}) + 2R_2(\text{new})$

Iteration-2		C_j	-1	-2	-2	0	0
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2
S_1	0	-5	$\left(-\frac{1}{2}\right)$	0	0	1	$-\frac{1}{2}$
x_3	-2	$\frac{7}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	$-\frac{1}{4}$
$Z = -7$		Z_j	$-\frac{1}{2}$	**-1**	**-2**	**0**	$\frac{1}{2}$
		$C_j - Z_j$	$-\frac{1}{2}$	-1	0	0	$-\frac{1}{2}$
		Ratio $= \frac{C_j - Z_j}{S_1,j}$ and $S_1,j < 0$	1 ↑	---	---	---	1

Minimum negative X_B is -5 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is 1 and its column index is 1. So, the entering variable is x_1.

$\therefore$ The pivot element is $-\frac{1}{2}$.

Entering $= x_1$, Departing $= S_1$, Key Element $= -\frac{1}{2}$

$R_1(\text{new}) = R_1(\text{old}) \times -2$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{4} R_1(\text{new})$

Iteration-3		C_j	-1	-2	-2	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**
x_1	-1	10	1	0	0	-2	1
x_3	-2	1	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$-\frac{1}{2}$
$Z = -12$		Z_j	**-1**	**-1**	**-2**	**1**	**0**
		$C_j - Z_j$	0	-1	0	-1	0
		Ratio	---	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 10, x_2 = 0, x_3 = 1$

Max $Z = -12$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = x1 + 2x2
subject to
-2x1 - x2 <= -4
-x1 - 2x2 <= -7
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = x_1 + 2x_2$$

subject to

$$-2x_1 - x_2 \le -4$$
$$-x_1 - 2x_2 \le -7$$

and $x_1, x_2 \ge 0$;

In order to apply the dual simplex method, convert Min Z to Max Z
Problem is

$$\text{Max } Z = -x_1 - 2x_2$$

subject to

$$-2x_1 - x_2 \le -4$$
$$-x_1 - 2x_2 \le -7$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -x_1 - 2x_2 + 0S_1 + 0S_2$$

subject to

$$-2x_1 - x_2 + S_1 = -4$$
$$-x_1 - 2x_2 + S_2 = -7$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	-1	-2	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-4	-2	-1	1	0
	0	-7	**(-1)**	-2	0	1

S_2						
$Z = 0$		Z_j	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-1	-2	0	0
		Ratio $= \dfrac{C_j - Z_j}{S_2,j}$ and $S_2,j < 0$	1 ↑	1	---	---

Minimum negative X_B is -7 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is 1 and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is -1.

Entering $= x_1$, Departing $= S_2$, Key Element $= -1$

$R_2(\text{new}) = R_2(\text{old}) \div -1$

$R_1(\text{new}) = R_1(\text{old}) + 2R_2(\text{new})$

Iteration-2		C_j	-1	-2	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	10	0	3	1	-2
x_1	-1	7	1	2	0	-1
$Z = -7$		Z_j	**-1**	**-2**	**0**	**1**
		$C_j - Z_j$	0	0	0	-1
		Ratio	---	---	---	---

Since all $C_j - Z_j \le 0$ and all $X_{Bi} \ge 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = 7, x_2 = 0$

Max $Z = -7$

Solution is provided by AtoZmath.com

Print This Solution Close This Solution

Find solution using dual-simplex method
MIN Z = x1 + x2
subject to
x1 + 3x2 >= 6
2x1 + x2 >= 8
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = x_1 + x_2$$

subject to

$$x_1 + 3x_2 \ge 6$$
$$2x_1 + x_2 \ge 8$$

and $x_1, x_2 \ge 0;$

In order to apply the dual simplex method, convert Min Z to Max Z and all $\ge$ constraint to $\le$ constraint by multiply -1.

Problem is

$$\text{Max } Z = -x_1 - x_2$$

subject to

$$-x_1 - 3x_2 \le -6$$
$$-2x_1 - x_2 \le -8$$

and $x_1, x_2 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = -x_1 - x_2 + 0S_1 + 0S_2$$

subject to

$$-x_1 - 3x_2 + S_1 = -6$$
$$-2x_1 - x_2 + S_2 = -8$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	-1	-1	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-6	-1	-3	1	0

S_2	0	-8	**(-2)**	-1	0	1
$Z = 0$		Z_j	**0**	**0**	**0**	**0**
		$C_j - Z_j$	-1	-1	0	0
		Ratio $= \frac{C_j - Z_j}{S_2, j}$ and $S_2, j < 0$	$\frac{1}{2}$ ↑	1	---	---

Minimum negative X_B is -8 and its row index is 2. So, the leaving basis variable is S_2.

Minimum positive ratio is $\frac{1}{2}$ and its column index is 1. So, the entering variable is x_1.

∴ The pivot element is -2.

Entering $= x_1$, Departing $= S_2$, Key Element $= -2$

$R_2(\text{new}) = R_2(\text{old}) \div -2$

$R_1(\text{new}) = R_1(\text{old}) + R_2(\text{new})$

Iteration-2		C_j	-1	-1	0	0
B	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-2	0	$\left(-\frac{5}{2}\right)$	1	$-\frac{1}{2}$
x_1	-1	4	1	$\frac{1}{2}$	0	$-\frac{1}{2}$
$Z = -4$		Z_j	**-1**	$-\frac{1}{2}$	**0**	$\frac{1}{2}$
		$C_j - Z_j$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
		Ratio $= \frac{C_j - Z_j}{S_1, j}$ and $S_1, j < 0$	---	$\frac{1}{5}$ ↑	---	1

Minimum negative X_B is -2 and its row index is 1. So, the leaving basis variable is S_1.

Minimum positive ratio is $\frac{1}{5}$ and its column index is 2. So, the entering variable is x_2.

∴ The pivot element is $-\frac{5}{2}$.

Entering $= x_2$, Departing $= S_1$, Key Element $= -\frac{5}{2}$

$R_1(\text{new}) = R_1(\text{old}) \times -\frac{2}{5}$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{2} R_1(\text{new})$

Iteration-3		C_j	-1	-1	0	0
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**
x_2	-1	$\frac{4}{5}$	0	1	$-\frac{2}{5}$	$\frac{1}{5}$
x_1	-1	$\frac{18}{5}$	1	0	$\frac{1}{5}$	$-\frac{3}{5}$
$Z = -\frac{22}{5}$		**Z_j**	**-1**	**-1**	**$\frac{1}{5}$**	**$\frac{2}{5}$**
		$C_j - Z_j$	0	0	$-\frac{1}{5}$	$-\frac{2}{5}$
		Ratio	---	---	---	---

Since all $C_j - Z_j \leq 0$ and all $X_{Bi} \geq 0$ thus the current solution is the optimal solution.

Hence, optimal solution is arrived with value of variables as :
$x_1 = \frac{18}{5}, x_2 = \frac{4}{5}$

Max $Z = -\frac{22}{5}$

Solution is provided by AtoZmath.com