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Find solution using Simplex(BigM) method
MAX Z = 5x1 + x2
subject to
5x1 + 2x2 <= 20
x1 >= 3
x2 <= 5
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 5x_1 + x_2$$

subject to

$$5x_1 + 2x_2 \leq 20$$
$$x_1 \geq 3$$
$$x_2 \leq 5$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_1

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_3

After introducing slack,surplus,artificial variables

$$\text{Max } Z = 5x_1 + x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1$$

subject to

$$5x_1 + 2x_2 + S_1 = 20$$
$$x_1 - S_2 + A_1 = 3$$
$$x_2 + S_3 = 5$$

and $x_1, x_2, S_1, S_2, S_3, A_1 \geq 0$

Iteration-1		C_j	5	1	0	0	0	$-M$	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_1	MinRatio $\frac{X_B}{x_1}$
S_1	0	20	5	2	1	0	0	0	$\frac{20}{5} = 4$
A_1	$-M$	3	(1)	0	0	-1	0	1	$\frac{3}{1} = 3 \rightarrow$

S_2	0	5	0	1	0	0	1	0	---
$Z = 0$		Z_j	$-M$	0	0	M	0	$-M$	
		$C_j - Z_j$	$M + 5$ ↑	1	0	$-M$	0	0	

Positive maximum $C_j - Z_j$ is $M + 5$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 3 and its row index is 2. So, the leaving basis variable is A_1.

∴ The pivot element is 1.

Entering $= x_1$, Departing $= A_1$, Key Element $= 1$

$R_2(\text{new}) = R_2(\text{old})$

$R_1(\text{new}) = R_1(\text{old}) - 5R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old})$

Iteration-2		C_j	5	1	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{S_2}$
S_1	0	5	0	2	1	**(5)**	0	$\frac{5}{5} = 1$ →
x_1	5	3	1	0	0	-1	0	---
S_2	0	5	0	1	0	0	1	---
$Z = 15$		Z_j	**5**	**0**	**0**	**-5**	**0**	
		$C_j - Z_j$	0	1	0	5 ↑	0	

Positive maximum $C_j - Z_j$ is 5 and its column index is 4. So, the entering variable is S_2.

Minimum ratio is 1 and its row index is 1. So, the leaving basis variable is S_1.

∴ The pivot element is 5.

Entering $= S_2$, Departing $= S_1$, Key Element $= 5$

$R_1(\text{new}) = R_1(\text{old}) \div 5$

$R_2(\text{new}) = R_2(\text{old}) + R_1(\text{new})$

R_3(new) = R_3(old)

Iteration-3		C_j	5	1	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio**
S_2	0	1	0	0.4	0.2	1	0	
x_1	5	4	1	0.4	0.2	0	0	
S_2	0	5	0	1	0	0	1	
$Z = 20$		Z_j	**5**	**2**	**1**	**0**	**0**	
		$C_j - Z_j$	0	-1	-1	0	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 4, x_2 = 0$

Max $Z = 20$

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Find solution using Simplex(BigM) method
MIN Z = 3x1 + 8x2
subject to
x1 + x2 = 200
x1 <= 80
x2 >= 60
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 3x_1 + 8x_2$$

subject to

$$x_1 + x_2 = 200$$
$$x_1 \le 80$$
$$x_2 \ge 60$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' = ' we should add artificial variable A_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_1

3. As the constraint 3 is of type ' $\ge$ ' we should subtract surplus variable S_2 and add artificial variable A_2

After introducing slack,surplus,artificial variables

$$\text{Min } Z = 3x_1 + 8x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$$

subject to

$$x_1 + x_2 + A_1 = 200$$
$$x_1 + S_1 = 80$$
$$x_2 - S_2 + A_2 = 60$$

and $x_1, x_2, S_1, S_2, A_1, A_2 \ge 0$

Iteration-1		C_j	3	8	0	0	M	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	MinRatio $\frac{X_B}{x_2}$
A_1	M	200	1	1	0	0	1	0	$\frac{200}{1} = 200$
S_1	0	80	1	0	1	0	0	0	---

A_2	M	60	0	**(1)**	0	-1	0	1	$\frac{60}{1}=60 \rightarrow$
$Z = 0$		Z_j	M	$2M$	0	$-M$	M	M	
		$C_j - Z_j$	$-M+3$	$-2M+8 \uparrow$	0	M	0	0	

Negative minimum $C_j - Z_j$ is $-2M+8$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 60 and its row index is 3. So, the leaving basis variable is A_2.

∴ The pivot element is 1.

Entering $= x_2$, Departing $= A_2$, Key Element $= 1$

R_3(new) = R_3(old)

R_1(new) = R_1(old)-R_3(new)

R_2(new) = R_2(old)

Iteration-2		C_j	3	8	0	0	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	**MinRatio** $\frac{X_B}{x_1}$
A_1	M	140	1	0	0	1	1	$\frac{140}{1}=140$
S_1	0	80	**(1)**	0	1	0	0	$\frac{80}{1}=80 \rightarrow$
x_2	8	60	0	1	0	-1	0	---
$Z = 480$		Z_j	M	8	0	$M-8$	M	
		$C_j - Z_j$	$-M+3 \uparrow$	0	0	$-M+8$	0	

Negative minimum $C_j - Z_j$ is $-M+3$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 80 and its row index is 2. So, the leaving basis variable is S_1.

∴ The pivot element is 1.

Entering $= x_1$, Departing $= S_1$, Key Element $= 1$

R_2(new) = R_2(old)

R_1(new) = R_1(old)-R_2(new)

R_3(new) = R_3(old)

Iteration-3		C_j	3	8	0	0	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**A_1**	**MinRatio** $\frac{X_B}{S_2}$
A_1	M	60	0	0	-1	(1)	1	$\frac{60}{1} = 60 \rightarrow$
x_1	3	80	1	0	1	0	0	---
x_2	8	60	0	1	0	-1	0	---
$Z = 720$		**Z_j**	**3**	**8**	**$-M+3$**	**$M-8$**	**M**	
		$C_j - Z_j$	0	0	$M-3$	$-M+8 \uparrow$	0	

Negative minimum $C_j - Z_j$ is $-M + 8$ and its column index is 4. So, the entering variable is S_2.

Minimum ratio is 60 and its row index is 1. So, the leaving basis variable is A_1.

∴ The pivot element is 1.

Entering $= S_2$, Departing $= A_1$, Key Element $= 1$

R_1(new) = R_1(old)

R_2(new) = R_2(old)

R_3(new) = R_3(old)+R_1(new)

Iteration-4		C_j	3	8	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio**
S_2	0	60	0	0	-1	1	
x_1	3	80	1	0	1	0	
x_2	8	120	0	1	-1	0	
$Z = 1200$		**Z_j**	**3**	**8**	**-5**	**0**	
		$C_j - Z_j$	0	0	5	0	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 80, x_2 = 120$

Min $Z = 1200$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 2x2 + 3x3
subject to
2x1 + x2 + x3 <= 2
3x1 + 4x2 + 2x3 >= 8
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 2x_2 + 3x_3$$

subject to

$$2x_1 + x_2 + x_3 \leq 2$$
$$3x_1 + 4x_2 + 2x_3 \geq 8$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_1

After introducing slack,surplus,artificial variables

$$\text{Max } Z = 3x_1 + 2x_2 + 3x_3 + 0S_1 + 0S_2 - MA_1$$

subject to

$$2x_1 + x_2 + x_3 + S_1 = 2$$
$$3x_1 + 4x_2 + 2x_3 - S_2 + A_1 = 8$$

and $x_1, x_2, x_3, S_1, S_2, A_1 \geq 0$

Iteration-1		C_j	3	2	3	0	0	$-M$	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	A_1	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	2	2	1	1	1	0	0	$\frac{2}{1} = 2$
A_1	$-M$	8	3	(4)	2	0	-1	1	$\frac{8}{4} = 2 \rightarrow$
$Z = 0$		Z_j	$-3M$	$-4M$	$-2M$	0	M	$-M$	
		$C_j - Z_j$	$3M + 3$	$4M + 2 \uparrow$	$2M + 3$	0	$-M$	0	

Positive maximum $C_j - Z_j$ is $4M + 2$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 2 and its row index is 2. So, the leaving basis variable is A_1.

∴ The pivot element is 4.

Entering $= x_2$, Departing $= A_1$, Key Element $= 4$

$R_2(\text{new}) = R_2(\text{old}) \div 4$

$R_1(\text{new}) = R_1(\text{old}) - R_2(\text{new})$

Iteration-2		C_j	3	2	3	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**MinRatio** $\frac{X_B}{x_3}$
S_1	0	0	$\frac{5}{4}$	0	$\left(\frac{1}{2}\right)$	1	$\frac{1}{4}$	$\frac{0}{\frac{1}{2}} = 0 \rightarrow$
x_2	2	2	$\frac{3}{4}$	1	$\frac{1}{2}$	0	$-\frac{1}{4}$	$\frac{2}{\frac{1}{2}} = 4$
$Z = 4$		**Z_j**	$\frac{3}{2}$	**2**	**1**	**0**	$-\frac{1}{2}$	
		$C_j - Z_j$	$\frac{3}{2}$	0	$2 \uparrow$	0	$\frac{1}{2}$	

Positive maximum $C_j - Z_j$ is 2 and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 0 and its row index is 1. So, the leaving basis variable is S_1.

∴ The pivot element is $\frac{1}{2}$.

Entering $= x_3$, Departing $= S_1$, Key Element $= \frac{1}{2}$

$R_1(\text{new}) = R_1(\text{old}) \times 2$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{2} R_1(\text{new})$

Iteration-3		C_j	3	2	3	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**MinRatio**

x_3	3	0	$\frac{5}{2}$	0	1	2	$\frac{1}{2}$	
x_2	2	2	$-\frac{1}{2}$	1	0	-1	$-\frac{1}{2}$	
$Z = 4$		Z_j	$\mathbf{\frac{13}{2}}$	**2**	**3**	**4**	$\mathbf{\frac{1}{2}}$	
		$C_j - Z_j$	$-\frac{7}{2}$	0	0	-4	$-\frac{1}{2}$	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 2, x_3 = 0$

Max $Z = 4$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 6x2
subject to
x1 + x2 <= 20
4x1 + x2 >= 20
x1 + x2 >= 18
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 6x_2$$

subject to

$$x_1 + x_2 \leq 20$$
$$4x_1 + x_2 \geq 20$$
$$x_1 + x_2 \geq 18$$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_1

3. As the constraint 3 is of type ' $\geq$ ' we should subtract surplus variable S_3 and add artificial variable A_2

After introducing slack,surplus,artificial variables

$$\text{Max } Z = 3x_1 + 6x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_2$$

subject to

$$x_1 + x_2 + S_1 = 20$$
$$4x_1 + x_2 - S_2 + A_1 = 20$$
$$x_1 + x_2 - S_3 + A_2 = 18$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2 \geq 0$

Iteration-1		C_j	3	6	0	0	0	$-M$	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	20	1	1	1	0	0	0	0	$\frac{20}{1} = 20$
A_1	$-M$	20	**(4)**	1	0	-1	0	1	0	$\frac{20}{4} = 5 \rightarrow$

A_2	$-M$	18	1	1	0	0	-1	0	1	$\frac{18}{1} = 18$
$Z = 0$		Z_j	$-5M$	$-2M$	0	M	M	$-M$	$-M$	
		$C_j - Z_j$	$5M + 3$ ↑	$2M + 6$	0	$-M$	$-M$	0	0	

Positive maximum $C_j - Z_j$ is $5M + 3$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 5 and its row index is 2. So, the leaving basis variable is A_1.

∴ The pivot element is 4.

Entering $= x_1$, Departing $= A_1$, Key Element $= 4$

$R_2(\text{new}) = R_2(\text{old}) \div 4$

$R_1(\text{new}) = R_1(\text{old}) - R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - R_2(\text{new})$

Iteration-2		C_j	3	6	0	0	0	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_2**	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	15	0	$\frac{3}{4}$	1	$\frac{1}{4}$	0	0	$\frac{15}{\frac{3}{4}} = 20$
x_1	3	5	1	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	$\frac{5}{\frac{1}{4}} = 20$
A_2	$-M$	13	0	$\left(\frac{3}{4}\right)$	0	$\frac{1}{4}$	-1	1	$\frac{13}{\frac{3}{4}} = \frac{52}{3}$ →
$Z = 15$		Z_j	3	$-\frac{3M}{4} + \frac{3}{4}$	0	$-\frac{M}{4} - \frac{3}{4}$	M	$-M$	
		$C_j - Z_j$	0	$\frac{3M}{4} + \frac{21}{4}$ ↑	0	$\frac{M}{4} + \frac{3}{4}$	$-M$	0	

Positive maximum $C_j - Z_j$ is $\frac{3M}{4} + \frac{21}{4}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{52}{3}$ and its row index is 3. So, the leaving basis variable is A_2.

$\therefore$ The pivot element is $\frac{3}{4}$.

Entering $= x_2$, Departing $= A_2$, Key Element $= \frac{3}{4}$

$R_3(\text{new}) = R_3(\text{old}) \times \frac{4}{3}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{3}{4} R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{4} R_3(\text{new})$

Iteration-3		C_j	3	6	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{S_3}$
S_1	0	2	0	0	1	0	**(1)**	$\frac{2}{1} = 2 \rightarrow$
x_1	3	$\frac{2}{3}$	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{\frac{2}{3}}{\frac{1}{3}} = 2$
x_2	6	$\frac{52}{3}$	0	1	0	$\frac{1}{3}$	$-\frac{4}{3}$	---
$Z = 106$		Z_j	**3**	**6**	**0**	**1**	**-7**	
		$C_j - Z_j$	0	0	0	-1	7 $\uparrow$	

Positive maximum $C_j - Z_j$ is 7 and its column index is 5. So, the entering variable is S_3.

Minimum ratio is 2 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 1.

Entering $= S_3$, Departing $= S_1$, Key Element $= 1$

$R_1(\text{new}) = R_1(\text{old})$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{3} R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + \frac{4}{3} R_1(\text{new})$

Iteration-4		C_j	3	6	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**MinRatio**
S_3	0	2	0	0	1	0	1	
x_1	3	0	1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	0	
x_2	6	20	0	1	$\frac{4}{3}$	$\frac{1}{3}$	0	
$Z = 120$		Z_j	**3**	**6**	**7**	**1**	**0**	
		$C_j - Z_j$	0	0	-7	-1	0	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 20$

Max $Z = 120$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + x2
subject to
4x1 + x2 = 4
5x1 + 3x2 >= 7
3x1 + 2x2 <= 6
and x1,x2 >= 0

Solution:
Problem is

Max $Z = 3x_1 + x_2$

subject to

$4x_1 + x_2 = 4$

$5x_1 + 3x_2 \geq 7$

$3x_1 + 2x_2 \leq 6$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' = ' we should add artificial variable A_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_1 and add artificial variable A_2

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack,surplus,artificial variables

Max $Z = 3x_1 + x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

subject to

$4x_1 + x_2 + A_1 = 4$

$5x_1 + 3x_2 - S_1 + A_2 = 7$

$3x_1 + 2x_2 + S_2 = 6$

and $x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Iteration-1		C_j	3	1	0	0	$-M$	$-M$	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	**MinRatio** $\frac{X_B}{x_1}$
A_1	$-M$	4	**(4)**	1	0	0	1	0	$\frac{4}{4} = 1 \rightarrow$
A_2	$-M$	7	5	3	-1	0	0	1	$\frac{7}{5} = \frac{7}{5}$

S_1	0	6	3	2	0	1	0	0	$\frac{6}{3} = 2$
$Z = 0$		Z_j	$-9M$	$-4M$	M	0	$-M$	$-M$	
		$C_j - Z_j$	$9M + 3 \uparrow$	$4M + 1$	$-M$	0	0	0	

Positive maximum $C_j - Z_j$ is $9M + 3$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 1 and its row index is 1. So, the leaving basis variable is A_1.

$\therefore$ The pivot element is 4.

Entering $= x_1$, Departing $= A_1$, Key Element $= 4$

$R_1(\text{new}) = R_1(\text{old}) \div 4$

$R_2(\text{new}) = R_2(\text{old}) - 5R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - 3R_1(\text{new})$

Iteration-2		C_j	3	1	0	0	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**A_2**	**MinRatio** $\frac{X_B}{x_2}$
x_1	3	1	1	$\frac{1}{4}$	0	0	0	$\frac{1}{\frac{1}{4}} = 4$
A_2	$-M$	2	0	$\left(\frac{7}{4}\right)$	-1	0	1	$\frac{2}{\frac{7}{4}} = \frac{8}{7} \rightarrow$
S_1	0	3	0	$\frac{5}{4}$	0	1	0	$\frac{3}{\frac{5}{4}} = \frac{12}{5}$
$Z = 3$		Z_j	3	$-\frac{7M}{4} + \frac{3}{4}$	M	0	$-M$	
		$C_j - Z_j$	0	$\frac{7M}{4} + \frac{1}{4} \uparrow$	$-M$	0	0	

Positive maximum $C_j - Z_j$ is $\frac{7M}{4} + \frac{1}{4}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{8}{7}$ and its row index is 2. So, the leaving basis variable is A_2.

$\therefore$ The pivot element is $\frac{7}{4}$.

Entering $= x_2$, Departing $= A_2$, Key Element $= \frac{7}{4}$

$R_2(\text{new}) = R_2(\text{old}) \times \frac{4}{7}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{4} R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{5}{4} R_2(\text{new})$

Iteration-3		C_j	3	1	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio** $\frac{X_B}{S_1}$
x_1	3	$\frac{5}{7}$	1	0	$\frac{1}{7}$	0	$\frac{\frac{5}{7}}{\frac{1}{7}} = 5$
x_2	1	$\frac{8}{7}$	0	1	$-\frac{4}{7}$	0	---
S_1	0	$\frac{11}{7}$	0	0	$\left(\frac{5}{7}\right)$	1	$\frac{\frac{11}{7}}{\frac{5}{7}} = \frac{11}{5} \rightarrow$
$Z = \frac{23}{7}$		Z_j	3	1	$-\frac{1}{7}$	0	
		$C_j - Z_j$	0	0	$\frac{1}{7} \uparrow$	0	

Positive maximum $C_j - Z_j$ is $\frac{1}{7}$ and its column index is 3. So, the entering variable is S_1.

Minimum ratio is $\frac{11}{5}$ and its row index is 3. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is $\frac{5}{7}$.

Entering $= S_1$, Departing $= S_1$, Key Element $= \frac{5}{7}$

$R_3(\text{new}) = R_3(\text{old}) \times \frac{7}{5}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{7} R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) + \frac{4}{7} R_3(\text{new})$

Iteration-4		C_j	3	1	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio**
x_1	3	$\frac{2}{5}$	1	0	0	$-\frac{1}{5}$	
x_2	1	$\frac{12}{5}$	0	1	0	$\frac{4}{5}$	
S_1	0	$\frac{11}{5}$	0	0	1	$\frac{7}{5}$	
$Z = \frac{18}{5}$		Z_j	**3**	**1**	**0**	**$\frac{1}{5}$**	
		$C_j - Z_j$	0	0	0	$-\frac{1}{5}$	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = \frac{2}{5}, x_2 = \frac{12}{5}$

Max $Z = \frac{18}{5}$

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Find solution using Simplex(BigM) method
MAX Z = 50x1 + 30x2
subject to
3x1 + 2x2 <= 34
x1 + x2 >= 12
3x1 + 2x2 >= 18
and x1,x2 >= 0

Solution:
Problem is

$\text{Max } Z = 50x_1 + 30x_2$

subject to

$3x_1 + 2x_2 \leq 34$

$x_1 + x_2 \geq 12$

$3x_1 + 2x_2 \geq 18$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_1

3. As the constraint 3 is of type ' $\geq$ ' we should subtract surplus variable S_3 and add artificial variable A_2

After introducing slack,surplus,artificial variables

$\text{Max } Z = 50x_1 + 30x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_2$

subject to

$$3x_1 + 2x_2 + S_1 = 34$$
$$x_1 + x_2 - S_2 + A_1 = 12$$
$$3x_1 + 2x_2 - S_3 + A_2 = 18$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2 \geq 0$

Iteration-1		C_j	50	30	0	0	0	$-M$	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	34	3	2	1	0	0	0	0	$\frac{34}{3} = \frac{34}{3}$
A_1	$-M$	12	1	1	0	-1	0	1	0	$\frac{12}{1} = 12$

A_2	$-M$	18	**(3)**	2	0	0	-1	0	1	$\frac{18}{3} = 6 \rightarrow$
$Z = 0$		Z_j	$-4M$	$-3M$	0	M	M	$-M$	$-M$	
		$C_j - Z_j$	$4M + 50 \uparrow$	$3M + 30$	0	$-M$	$-M$	0	0	

Positive maximum $C_j - Z_j$ is $4M + 50$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 6 and its row index is 3. So, the leaving basis variable is A_2.

$\therefore$ The pivot element is 3.

Entering $= x_1$, Departing $= A_2$, Key Element $= 3$

$R_3(\text{new}) = R_3(\text{old}) \div 3$

$R_1(\text{new}) = R_1(\text{old}) - 3R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - R_3(\text{new})$

Iteration-2		C_j	50	30	0	0	0	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**MinRatio** $\frac{X_B}{S_3}$
S_1	0	16	0	0	1	0	**(1)**	0	$\frac{16}{1} = 16 \rightarrow$
A_1	$-M$	6	0	$\frac{1}{3}$	0	-1	$\frac{1}{3}$	1	$\frac{6}{\frac{1}{3}} = 18$
x_1	50	6	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	---
$Z = 300$		Z_j	50	$-\frac{M}{3} + \frac{100}{3}$	0	M	$-\frac{M}{3} - \frac{50}{3}$	$-M$	
		$C_j - Z_j$	0	$\frac{M}{3} - \frac{10}{3}$	0	$-M$	$\frac{M}{3} + \frac{50}{3} \uparrow$	0	

Positive maximum $C_j - Z_j$ is $\frac{M}{3} + \frac{50}{3}$ and its column index is 5. So, the entering variable is S_3.

Minimum ratio is 16 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 1.

Entering $= S_3$, Departing $= S_1$, Key Element $= 1$

$R_1(\text{new}) = R_1(\text{old})$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{3} R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + \frac{1}{3} R_1(\text{new})$

Iteration-3		C_j	50	30	0	0	0	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**MinRatio** $\frac{X_B}{x_2}$
S_3	0	16	0	0	1	0	1	0	---
A_1	$-M$	$\frac{2}{3}$	0	$\left(\frac{1}{3}\right)$	$-\frac{1}{3}$	-1	0	1	$\frac{\frac{2}{3}}{\frac{1}{3}} = 2 \rightarrow$
x_1	50	$\frac{34}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	$\frac{\frac{34}{3}}{\frac{2}{3}} = 17$
$Z = \frac{1700}{3}$		Z_j	50	$-\frac{M}{3} + \frac{100}{3}$	$\frac{M}{3} + \frac{50}{3}$	M	0	$-M$	
		$C_j - Z_j$	0	$\frac{M}{3} - \frac{10}{3} \uparrow$	$-\frac{M}{3} - \frac{50}{3}$	$-M$	0	0	

Positive maximum $C_j - Z_j$ is $\frac{M}{3} - \frac{10}{3}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 2 and its row index is 2. So, the leaving basis variable is A_1.

$\therefore$ The pivot element is $\frac{1}{3}$.

Entering $= x_2$, Departing $= A_1$, Key Element $= \frac{1}{3}$

$R_2(\text{new}) = R_2(\text{old}) \times 3$

$R_1(\text{new}) = R_1(\text{old})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{2}{3} R_2(\text{new})$

Iteration-4		C_j	50	30	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio**
S_3	0	16	0	0	1	0	1	
x_2	30	2	0	1	-1	-3	0	
x_1	50	10	1	0	1	2	0	
$Z = 560$		Z_j	**50**	**30**	**20**	**10**	**0**	
		$C_j - Z_j$	0	0	-20	-10	0	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 10, x_2 = 2$

Max $Z = 560$

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Find solution using Simplex(BigM) method
MIN Z = 2x1 + 10x2
subject to
x1 + 2x2 <= 40
3x1 + x2 >= 30
4x1 + 3x2 >= 64
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 2x_1 + 10x_2$$

subject to

$$x_1 + 2x_2 \le 40$$
$$3x_1 + x_2 \ge 30$$
$$4x_1 + 3x_2 \ge 64$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\ge$ ' we should subtract surplus variable S_2 and add artificial variable A_1

3. As the constraint 3 is of type ' $\ge$ ' we should subtract surplus variable S_3 and add artificial variable A_2

After introducing slack,surplus,artificial variables

$$\text{Min } Z = 2x_1 + 10x_2 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2$$

subject to

$$x_1 + 2x_2 + S_1 = 40$$
$$3x_1 + x_2 - S_2 + A_1 = 30$$
$$4x_1 + 3x_2 - S_3 + A_2 = 64$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2 \ge 0$

Iteration-1		C_j	2	10	0	0	0	M	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	MinRatio $\frac{X_B}{x_1}$
S_1	0	40	1	2	1	0	0	0	0	$\frac{40}{1} = 40$
A_1	M	30	(3)	1	0	-1	0	1	0	$\frac{30}{3} = 10 \rightarrow$

A_2	M	64	4	3	0	0	-1	0	1	$\frac{64}{4} = 16$
$Z = 0$		Z_j	$7M$	$4M$	0	$-M$	$-M$	M	M	
		$C_j - Z_j$	$-7M + 2$ ↑	$-4M + 10$	0	M	M	0	0	

Negative minimum $C_j - Z_j$ is $-7M + 2$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 10 and its row index is 2. So, the leaving basis variable is A_1.

∴ The pivot element is 3.

Entering $= x_1$, Departing $= A_1$, Key Element $= 3$

$R_2(\text{new}) = R_2(\text{old}) \div 3$

$R_1(\text{new}) = R_1(\text{old}) - R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - 4R_2(\text{new})$

Iteration-2		C_j	2	10	0	0	0	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_2**	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	30	0	$\frac{5}{3}$	1	$\frac{1}{3}$	0	0	$\frac{30}{\frac{5}{3}} = 18$
x_1	2	10	1	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	0	$\frac{10}{\frac{1}{3}} = 30$
A_2	M	24	0	$\left(\frac{5}{3}\right)$	0	$\frac{4}{3}$	-1	1	$\frac{24}{\frac{5}{3}} = \frac{72}{5}$ →
$Z = 20$		Z_j	2	$\frac{5M}{3} + \frac{2}{3}$	0	$\frac{4M}{3} - \frac{2}{3}$	$-M$	M	
		$C_j - Z_j$	0	$-\frac{5M}{3} + \frac{28}{3}$ ↑	0	$-\frac{4M}{3} + \frac{2}{3}$	M	0	

Negative minimum $C_j - Z_j$ is $-\frac{5M}{3} + \frac{28}{3}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{72}{5}$ and its row index is 3. So, the leaving basis variable is A_2.

$\therefore$ The pivot element is $\frac{5}{3}$.

Entering $= x_2$, Departing $= A_2$, Key Element $= \frac{5}{3}$

$R_3(\text{new}) = R_3(\text{old}) \times \frac{3}{5}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{5}{3} R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{3} R_3(\text{new})$

Iteration-3		C_j	2	10	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{S_2}$
S_1	0	6	0	0	1	-1	1	---
x_1	2	$\frac{26}{5}$	1	0	0	$-\frac{3}{5}$	$\frac{1}{5}$	---
x_2	10	$\frac{72}{5}$	0	1	0	$\left(\frac{4}{5}\right)$	$-\frac{3}{5}$	$\frac{\frac{72}{5}}{\frac{4}{5}} = 18 \rightarrow$
$Z = \frac{772}{5}$		Z_j	2	10	0	$\frac{34}{5}$	$-\frac{28}{5}$	
		$C_j - Z_j$	0	0	0	$-\frac{34}{5} \uparrow$	$\frac{28}{5}$	

Negative minimum $C_j - Z_j$ is $-\frac{34}{5}$ and its column index is 4. So, the entering variable is S_2.

Minimum ratio is 18 and its row index is 3. So, the leaving basis variable is x_2.

$\therefore$ The pivot element is $\frac{4}{5}$.

Entering $= S_2$, Departing $= x_2$, Key Element $= \frac{4}{5}$

$R_3(\text{new}) = R_3(\text{old}) \times \frac{5}{4}$

$R_1(\text{new}) = R_1(\text{old}) + R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) + \frac{3}{5} R_3(\text{new})$

Iteration-4		C_j	2	10	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio**
S_1	0	24	0	$\frac{5}{4}$	1	0	$\frac{1}{4}$	
x_1	2	16	1	$\frac{3}{4}$	0	0	$-\frac{1}{4}$	
S_2	0	18	0	$\frac{5}{4}$	0	1	$-\frac{3}{4}$	
$Z = 32$		Z_j	**2**	$\frac{3}{2}$	**0**	**0**	$-\frac{1}{2}$	
		$C_j - Z_j$	0	$\frac{17}{2}$	0	0	$\frac{1}{2}$	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 16, x_2 = 0$

Min $Z = 32$

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Find solution using Simplex(BigM) method
MIN Z = 3x1 + 2x2
subject to
5x1 + x2 >= 10
2x1 + 2x2 >= 12
x1 + 4x2 >= 12
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 3x_1 + 2x_2$$

subject to

$$5x_1 + x_2 \geq 10$$
$$2x_1 + 2x_2 \geq 12$$
$$x_1 + 4x_2 \geq 12$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' ≥ ' we should subtract surplus variable S_1 and add artificial variable A_1

2. As the constraint 2 is of type ' ≥ ' we should subtract surplus variable S_2 and add artificial variable A_2

3. As the constraint 3 is of type ' ≥ ' we should subtract surplus variable S_3 and add artificial variable A_3

After introducing surplus,artificial variables

$$\text{Min } Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2 + MA_3$$

subject to

$$5x_1 + x_2 - S_1 + A_1 = 10$$
$$2x_1 + 2x_2 - S_2 + A_2 = 12$$
$$x_1 + 4x_2 - S_3 + A_3 = 12$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2, A_3 \geq 0$

Iteration-1		C_j	3	2	0	0	0	M	M	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3	MinRatio $\frac{X_B}{x_1}$
A_1	M	10	(5)	1	-1	0	0	1	0	0	$\frac{10}{5} = 2 \rightarrow$
A_2	M	12	2	2	0	-1	0	0	1	0	$\frac{12}{2} = 6$

A_3	M	12	1	4	0	0	-1	0	0	1	$\frac{12}{1} = 12$
$\boldsymbol{Z = 0}$		Z_j	$\boldsymbol{8M}$	$\boldsymbol{7M}$	$\boldsymbol{-M}$	$\boldsymbol{-M}$	$\boldsymbol{-M}$	$\boldsymbol{M}$	$\boldsymbol{M}$	$\boldsymbol{M}$	
		$C_j - Z_j$	$-8M + 3$ ↑	$-7M + 2$	M	M	M	0	0	0	

Negative minimum $C_j - Z_j$ is $-8M + 3$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 2 and its row index is 1. So, the leaving basis variable is A_1.

∴ The pivot element is 5.

Entering $= x_1$, Departing $= A_1$, Key Element $= 5$

$R_1(\text{new}) = R_1(\text{old}) \div 5$

$R_2(\text{new}) = R_2(\text{old}) - 2R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - R_1(\text{new})$

Iteration-2		C_j	3	2	0	0	0	M	M	
$\boldsymbol{B}$	$\boldsymbol{C_B}$	$\boldsymbol{X_B}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	$\boldsymbol{S_3}$	$\boldsymbol{A_2}$	$\boldsymbol{A_3}$	**MinRatio** $\frac{\boldsymbol{X_B}}{\boldsymbol{x_2}}$
x_1	3	2	1	$\frac{1}{5}$	$-\frac{1}{5}$	0	0	0	0	$\frac{2}{\frac{1}{5}} = 10$
A_2	M	8	0	$\frac{8}{5}$	$\frac{2}{5}$	-1	0	1	0	$\frac{8}{\frac{8}{5}} = 5$
A_3	M	10	0	$\left(\boldsymbol{\frac{19}{5}}\right)$	$\frac{1}{5}$	0	-1	0	1	$\frac{10}{\frac{19}{5}} = \frac{50}{19}$ →
$\boldsymbol{Z = 6}$		Z_j	3	$\boldsymbol{\frac{27M}{5} + \frac{3}{5}}$	$\boldsymbol{\frac{3M}{5} - \frac{3}{5}}$	$\boldsymbol{-M}$	$\boldsymbol{-M}$	$\boldsymbol{M}$	$\boldsymbol{M}$	
		$C_j - Z_j$	0	$-\frac{27M}{5} + \frac{7}{5}$ ↑	$-\frac{3M}{5} + \frac{3}{5}$	M	M	0	0	

Negative minimum $C_j - Z_j$ is $-\frac{27M}{5} + \frac{7}{5}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{50}{19}$ and its row index is 3. So, the leaving basis variable is A_3.

∴ The pivot element is $\frac{19}{5}$.

Entering $= x_2$, Departing $= A_3$, Key Element $= \frac{19}{5}$

$R_3(\text{new}) = R_3(\text{old}) \times \frac{5}{19}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{5} R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - \frac{8}{5} R_3(\text{new})$

Iteration-3		C_j	3	2	0	0	0	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_2	**MinRatio** $\frac{X_B}{S_3}$
x_1	3	$\frac{28}{19}$	1	0	$-\frac{4}{19}$	0	$\frac{1}{19}$	0	$\frac{\frac{28}{19}}{\frac{1}{19}} = 28$
A_2	M	$\frac{72}{19}$	0	0	$\frac{6}{19}$	-1	$\left(\frac{8}{19}\right)$	1	$\frac{\frac{72}{19}}{\frac{8}{19}} = 9 \rightarrow$
x_2	2	$\frac{50}{19}$	0	1	$\frac{1}{19}$	0	$-\frac{5}{19}$	0	---
$Z = \frac{184}{19}$		Z_j	3	2	$\frac{6M}{19} - \frac{10}{19}$	$-M$	$\frac{8M}{19} - \frac{7}{19}$	M	
		$C_j - Z_j$	0	0	$-\frac{6M}{19} + \frac{10}{19}$	M	$-\frac{8M}{19} + \frac{7}{19} \uparrow$	0	

Negative minimum $C_j - Z_j$ is $-\frac{8M}{19} + \frac{7}{19}$ and its column index is 5. So, the entering variable is S_3.

Minimum ratio is 9 and its row index is 2. So, the leaving basis variable is A_2.

∴ The pivot element is $\frac{8}{19}$.

Entering $= S_3$, Departing $= A_2$, Key Element $= \frac{8}{19}$

$R_2(\text{new}) = R_2(\text{old}) \times \frac{19}{8}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{19} R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + \frac{5}{19} R_2(\text{new})$

Iteration-4		C_j	3	2	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio**
x_1	3	1	1	0	$-\frac{1}{4}$	$\frac{1}{8}$	0	
S_3	0	9	0	0	$\frac{3}{4}$	$-\frac{19}{8}$	1	
x_2	2	5	0	1	$\frac{1}{4}$	$-\frac{5}{8}$	0	
$Z = 13$		Z_j	**3**	**2**	$-\frac{1}{4}$	$-\frac{7}{8}$	**0**	
		$C_j - Z_j$	0	0	$\frac{1}{4}$	$\frac{7}{8}$	0	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 1, x_2 = 5$

Min $Z = 13$

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Find solution using Simplex(BigM) method
MIN Z = 5x1 + 3x2
subject to
2x1 + 4x2 <= 12
2x1 + 2x2 = 10
5x1 + 2x2 >= 10
and x1,x2 >= 0

Solution:
Problem is

$\text{Min } Z = 5x_1 + 3x_2$

subject to

$2x_1 + 4x_2 \le 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \ge 10$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' = ' we should add artificial variable A_1

3. As the constraint 3 is of type ' $\ge$ ' we should subtract surplus variable S_2 and add artificial variable A_2

After introducing slack,surplus,artificial variables

$\text{Min } Z = 5x_1 + 3x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

subject to

$2x_1 + 4x_2 + S_1 = 12$

$2x_1 + 2x_2 + A_1 = 10$

$5x_1 + 2x_2 - S_2 + A_2 = 10$

and $x_1, x_2, S_1, S_2, A_1, A_2 \ge 0$

Iteration-1		C_j	5	3	0	0	M	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	MinRatio $\frac{X_B}{x_1}$
S_1	0	12	2	4	1	0	0	0	$\frac{12}{2} = 6$
A_1	M	10	2	2	0	0	1	0	$\frac{10}{2} = 5$

A_2	M	10	**(5)**	2	0	-1	0	1	$\frac{10}{5} = 2 \rightarrow$
$Z = 0$		Z_j	$7M$	$4M$	0	$-M$	M	M	
		$C_j - Z_j$	$-7M + 5 \uparrow$	$-4M + 3$	0	M	0	0	

Negative minimum $C_j - Z_j$ is $-7M + 5$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 2 and its row index is 3. So, the leaving basis variable is A_2.

∴ The pivot element is 5.

Entering $= x_1$, Departing $= A_2$, Key Element $= 5$

$R_3(\text{new}) = R_3(\text{old}) \div 5$

$R_1(\text{new}) = R_1(\text{old}) - 2R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - 2R_3(\text{new})$

Iteration-2		C_j	5	3	0	0	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**A_1**	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	8	0	$\left(\frac{16}{5}\right)$	1	$\frac{2}{5}$	0	$\frac{8}{\frac{16}{5}} = \frac{5}{2} \rightarrow$
A_1	M	6	0	$\frac{6}{5}$	0	$\frac{2}{5}$	1	$\frac{6}{\frac{6}{5}} = 5$
x_1	5	2	1	$\frac{2}{5}$	0	$-\frac{1}{5}$	0	$\frac{2}{\frac{2}{5}} = 5$
$Z = 10$		Z_j	5	$\frac{6M}{5} + 2$	0	$\frac{2M}{5} - 1$	M	
		$C_j - Z_j$	0	$-\frac{6M}{5} + 1 \uparrow$	0	$-\frac{2M}{5} + 1$	0	

Negative minimum $C_j - Z_j$ is $-\frac{6M}{5} + 1$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{5}{2}$ and its row index is 1. So, the leaving basis variable is S_1.

∴ The pivot element is $\frac{16}{5}$.

Entering $= x_2$, Departing $= S_1$, Key Element $= \frac{16}{5}$

$R_1(\text{new}) = R_1(\text{old}) \times \frac{5}{16}$

$R_2(\text{new}) = R_2(\text{old}) - \frac{6}{5} R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{2}{5} R_1(\text{new})$

Iteration-3		C_j	5	3	0	0	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**A_1**	**MinRatio $\frac{X_B}{S_2}$**
x_2	3	$\frac{5}{2}$	0	1	$\frac{5}{16}$	$\frac{1}{8}$	0	$\frac{\frac{5}{2}}{\frac{1}{8}} = 20$
A_1	M	3	0	0	$-\frac{3}{8}$	$\left(\frac{1}{4}\right)$	1	$\frac{3}{\frac{1}{4}} = 12 \rightarrow$
x_1	5	1	1	0	$-\frac{1}{8}$	$-\frac{1}{4}$	0	---
$Z = \frac{25}{2}$		Z_j	5	3	$-\frac{3M}{8} + \frac{5}{16}$	$\frac{M}{4} - \frac{7}{8}$	M	
		$C_j - Z_j$	0	0	$\frac{3M}{8} - \frac{5}{16}$	$-\frac{M}{4} + \frac{7}{8} \uparrow$	0	

Negative minimum $C_j - Z_j$ is $-\frac{M}{4} + \frac{7}{8}$ and its column index is 4. So, the entering variable is S_2.

Minimum ratio is 12 and its row index is 2. So, the leaving basis variable is A_1.

∴ The pivot element is $\frac{1}{4}$.

Entering $= S_2$, Departing $= A_1$, Key Element $= \frac{1}{4}$

$R_2(\text{new}) = R_2(\text{old}) \times 4$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{8} R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + \frac{1}{4} R_2(\text{new})$

Iteration-4		C_j	5	3	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio**
x_2	3	1	0	1	$\frac{1}{2}$	0	
S_2	0	12	0	0	$-\frac{3}{2}$	1	
x_1	5	4	1	0	$-\frac{1}{2}$	0	
$Z = 23$		Z_j	**5**	**3**	**-1**	**0**	
		$C_j - Z_j$	0	0	1	0	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 4, x_2 = 1$

Min $Z = 23$

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Find solution using Simplex(BigM) method
MIN Z = 8x1 + 6x2
subject to
3x1 + 8x2 <= 96
2x1 + x2 >= 10
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 8x_1 + 6x_2$$

subject to

$$3x_1 + 8x_2 \le 96$$
$$2x_1 + x_2 \ge 10$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\ge$ ' we should subtract surplus variable S_2 and add artificial variable A_1

After introducing slack,surplus,artificial variables

$$\text{Min } Z = 8x_1 + 6x_2 + 0S_1 + 0S_2 + MA_1$$

subject to

$$3x_1 + 8x_2 + S_1 = 96$$
$$2x_1 + x_2 - S_2 + A_1 = 10$$

and $x_1, x_2, S_1, S_2, A_1 \ge 0$

Iteration-1		C_j	8	6	0	0	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	96	3	8	1	0	0	$\frac{96}{3} = 32$
A_1	M	10	**(2)**	1	0	-1	1	$\frac{10}{2} = 5 \rightarrow$
$Z = 0$		Z_j	$2M$	M	0	$-M$	M	
		$C_j - Z_j$	$-2M + 8 \uparrow$	$-M + 6$	0	M	0	

Negative minimum $C_j - Z_j$ is $-2M + 8$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 5 and its row index is 2. So, the leaving basis variable is A_1.

$\therefore$ The pivot element is 2.

Entering $= x_1$, Departing $= A_1$, Key Element $= 2$

$R_2(\text{new}) = R_2(\text{old}) \div 2$

$R_1(\text{new}) = R_1(\text{old}) - 3R_2(\text{new})$

Iteration-2		C_j	8	6	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio**
S_1	0	81	0	$\frac{13}{2}$	1	$\frac{3}{2}$	
x_1	8	5	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	
$Z = 40$		Z_j	**8**	**4**	**0**	**-4**	
		$C_j - Z_j$	0	2	0	4	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 5, x_2 = 0$

Min $Z = 40$

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Find solution using Simplex(BigM) method
MIN Z = 20x1 + 10x2
subject to
x1 + 2x2 <= 40
3x1 + x2 >= 30
4x1 + 3x2 >= 60
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 20x_1 + 10x_2$$

subject to

$$x_1 + 2x_2 \leq 40$$
$$3x_1 + x_2 \geq 30$$
$$4x_1 + 3x_2 \geq 60$$

and $x_1, x_2 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_1

3. As the constraint 3 is of type ' $\geq$ ' we should subtract surplus variable S_3 and add artificial variable A_2

After introducing slack,surplus,artificial variables

$$\text{Min } Z = 20x_1 + 10x_2 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2$$

subject to

$$x_1 + 2x_2 + S_1 = 40$$
$$3x_1 + x_2 - S_2 + A_1 = 30$$
$$4x_1 + 3x_2 - S_3 + A_2 = 60$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2 \geq 0$

Iteration-1		C_j	20	10	0	0	0	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	40	1	2	1	0	0	0	0	$\frac{40}{1} = 40$
A_1	M	30	**(3)**	1	0	-1	0	1	0	$\frac{30}{3} = 10 \rightarrow$

A_2	M	60	4	3	0	0	-1	0	1	$\frac{60}{4} = 15$
$Z = 0$		Z_j	$7M$	$4M$	0	$-M$	$-M$	M	M	
		$C_j - Z_j$	$-7M + 20$ ↑	$-4M + 10$	0	M	M	0	0	

Negative minimum $C_j - Z_j$ is $-7M + 20$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 10 and its row index is 2. So, the leaving basis variable is A_1.

∴ The pivot element is 3.

Entering $= x_1$, Departing $= A_1$, Key Element $= 3$

$R_2(\text{new}) = R_2(\text{old}) \div 3$

$R_1(\text{new}) = R_1(\text{old}) - R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - 4R_2(\text{new})$

Iteration-2		C_j	20	10	0	0	0	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_2**	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	30	0	$\frac{5}{3}$	1	$\frac{1}{3}$	0	0	$\frac{30}{\frac{5}{3}} = 18$
x_1	20	10	1	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	0	$\frac{10}{\frac{1}{3}} = 30$
A_2	M	20	0	$\left(\frac{5}{3}\right)$	0	$\frac{4}{3}$	-1	1	$\frac{20}{\frac{5}{3}} = 12$ →
$Z = 200$		Z_j	20	$\frac{5M}{3} + \frac{20}{3}$	0	$\frac{4M}{3} - \frac{20}{3}$	$-M$	M	
		$C_j - Z_j$	0	$-\frac{5M}{3} + \frac{10}{3}$ ↑	0	$-\frac{4M}{3} + \frac{20}{3}$	M	0	

Negative minimum $C_j - Z_j$ is $-\frac{5M}{3} + \frac{10}{3}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 12 and its row index is 3. So, the leaving basis variable is A_2.

∴ The pivot element is $\frac{5}{3}$.

Entering $= x_2$, Departing $= A_2$, Key Element $= \frac{5}{3}$

$R_3(\text{new}) = R_3(\text{old}) \times \frac{3}{5}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{5}{3} R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{3} R_3(\text{new})$

Iteration-3		C_j	20	10	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**MinRatio**
S_1	0	10	0	0	1	-1	1	
x_1	20	6	1	0	0	$-\frac{3}{5}$	$\frac{1}{5}$	
x_2	10	12	0	1	0	$\frac{4}{5}$	$-\frac{3}{5}$	
$Z = 240$		Z_j	**20**	**10**	**0**	**-4**	**-2**	
		$C_j - Z_j$	0	0	0	4	2	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 6, x_2 = 12$

Min $Z = 240$

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Find solution using Simplex(BigM) method
MIN Z = 200x1 + 400x2
subject to
x1 + x2 >= 200
x1 + 3x2 >= 100
x1 + 3x2 <= 35
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 200x_1 + 400x_2$$

subject to

$$x_1 + x_2 \geq 200$$

$$x_1 + 3x_2 \geq 100$$

$$x_1 + 3x_2 \leq 35$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\geq$ ' we should subtract surplus variable S_1 and add artificial variable A_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_2

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_3

After introducing slack,surplus,artificial variables

$$\text{Min } Z = 200x_1 + 400x_2 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2$$

subject to

$$x_1 + x_2 - S_1 + A_1 = 200$$

$$x_1 + 3x_2 - S_2 + A_2 = 100$$

$$x_1 + 3x_2 + S_3 = 35$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2 \geq 0$

Iteration-1		C_j	200	400	0	0	0	M	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	**MinRatio** $\frac{X_B}{x_2}$
A_1	M	200	1	1	-1	0	0	1	0	$\frac{200}{1} = 200$
A_2	M	100	1	3	0	-1	0	0	1	$\frac{100}{3} = \frac{100}{3}$

S_1	0	35	1	**(3)**	0	0	1	0	0	$\frac{35}{3} = \frac{35}{3} \rightarrow$
$Z = 0$		Z_j	$2M$	$4M$	$-M$	$-M$	0	M	M	
		$C_j - Z_j$	$-2M + 200$	$-4M + 400 \uparrow$	M	M	0	0	0	

Negative minimum $C_j - Z_j$ is $-4M + 400$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{35}{3}$ and its row index is 3. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 3.

Entering $= x_2$, Departing $= S_1$, Key Element $= 3$

$R_3(\text{new}) = R_3(\text{old}) \div 3$

$R_1(\text{new}) = R_1(\text{old}) - R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - 3R_3(\text{new})$

Iteration-2		C_j	200	400	0	0	0	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**MinRatio** $\frac{X_B}{x_1}$
A_1	M	$\frac{565}{3}$	$\frac{2}{3}$	0	-1	0	$-\frac{1}{3}$	1	0	$\frac{\frac{565}{3}}{\frac{2}{3}} = \frac{565}{2}$
A_2	M	65	0	0	0	-1	-1	0	1	---
x_2	400	$\frac{35}{3}$	$\left(\frac{1}{3}\right)$	1	0	0	$\frac{1}{3}$	0	0	$\frac{\frac{35}{3}}{\frac{1}{3}} = 35 \rightarrow$
$Z = \frac{14000}{3}$		Z_j	$\frac{2M}{3} + \frac{400}{3}$	400	$-M$	$-M$	$-\frac{4M}{3} + \frac{400}{3}$	M	M	
		$C_j - Z_j$	$-\frac{2M}{3} + \frac{200}{3} \uparrow$	0	M	M	$\frac{4M}{3} - \frac{400}{3}$	0	0	

Negative minimum $C_j - Z_j$ is $-\frac{2M}{3} + \frac{200}{3}$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 35 and its row index is 3. So, the leaving basis variable is x_2.

$\therefore$ The pivot element is $\frac{1}{3}$.

Entering $= x_1$, Departing $= x_2$, Key Element $= \frac{1}{3}$

$R_3(\text{new}) = R_3(\text{old}) \times 3$

$R_1(\text{new}) = R_1(\text{old}) - \frac{2}{3} R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old})$

Iteration-3		C_j	200	400	0	0	0	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**MinRatio**
A_1	M	165	0	-2	-1	0	-1	1	0	
A_2	M	65	0	0	0	-1	-1	0	1	
x_1	200	35	1	3	0	0	1	0	0	
$Z = 7000$		**Z_j**	**200**	**$-2M + 600$**	**$-M$**	**$-M$**	**$-2M + 200$**	**M**	**M**	
		$C_j - Z_j$	0	$2M - 200$	M	M	$2M - 200$	0	0	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 35, x_2 = 0$

Min $Z = 7000$

But this solution is not feasible
because the final solution violates the 1st constraint $x_1 + x_2 \geq 200$.

and the artificial variable A_1 appears in the basis with positive value 165

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Find solution using Simplex(BigM) method
MIN Z = 600x1 + 400x2
subject to
15x1 + 15x2 >= 200
3x1 + x2 >= 40
2x1 + 5x2 >= 44
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 600x_1 + 400x_2$$

subject to

$$15x_1 + 15x_2 \geq 200$$
$$3x_1 + x_2 \geq 40$$
$$2x_1 + 5x_2 \geq 44$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\geq$ ' we should subtract surplus variable S_1 and add artificial variable A_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_2

3. As the constraint 3 is of type ' $\geq$ ' we should subtract surplus variable S_3 and add artificial variable A_3

After introducing surplus,artificial variables

$$\text{Min } Z = 600x_1 + 400x_2 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2 + MA_3$$

subject to

$$15x_1 + 15x_2 - S_1 + A_1 = 200$$
$$3x_1 + x_2 - S_2 + A_2 = 40$$
$$2x_1 + 5x_2 - S_3 + A_3 = 44$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2, A_3 \geq 0$

Iteration-1		C_j	600	400	0	0	0	M	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**A_3**	**MinRatio** $\frac{X_B}{x_2}$
A_1	M	200	15	15	-1	0	0	1	0	0	$\frac{200}{15} = \frac{40}{3}$
A_2	M	40	3	1	0	-1	0	0	1	0	$\frac{40}{1} = 40$

A_3	M	44	2	**(5)**	0	0	-1	0	0	1	$\frac{44}{5} = \frac{44}{5} \rightarrow$
$Z = 0$		Z_j	$20M$	$21M$	$-M$	$-M$	$-M$	M	M	M	
		$C_j - Z_j$	$-20M + 600$	$-21M + 400 \uparrow$	M	M	M	0	0	0	

Negative minimum $C_j - Z_j$ is $-21M + 400$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{44}{5}$ and its row index is 3. So, the leaving basis variable is A_3.

$\therefore$ The pivot element is 5.

Entering $= x_2$, Departing $= A_3$, Key Element $= 5$

$R_3(\text{new}) = R_3(\text{old}) \div 5$

$R_1(\text{new}) = R_1(\text{old}) - 15R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - R_3(\text{new})$

Iteration-2		C_j	600	400	0	0	0	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**MinRatio** $\frac{X_B}{x_1}$
A_1	M	68	**(9)**	0	-1	0	3	1	0	$\frac{68}{9} = \frac{68}{9} \rightarrow$
A_2	M	$\frac{156}{5}$	$\frac{13}{5}$	0	0	-1	$\frac{1}{5}$	0	1	$\frac{\frac{156}{5}}{\frac{13}{5}} = 12$
x_2	400	$\frac{44}{5}$	$\frac{2}{5}$	1	0	0	$-\frac{1}{5}$	0	0	$\frac{\frac{44}{5}}{\frac{2}{5}} = 22$
$Z = 3520$		Z_j	$\frac{58M}{5} + 160$	400	$-M$	$-M$	$\frac{16M}{5} - 80$	M	M	
		$C_j - Z_j$	$-\frac{58M}{5} + 440 \uparrow$	0	M	M	$-\frac{16M}{5} + 80$	0	0	

Negative minimum $C_j - Z_j$ is $-\frac{58M}{5} + 440$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is $\frac{68}{9}$ and its row index is 1. So, the leaving basis variable is A_1.

∴ The pivot element is 9.

Entering $= x_1$, Departing $= A_1$, Key Element $= 9$

$R_1(\text{new}) = R_1(\text{old}) \div 9$

$R_2(\text{new}) = R_2(\text{old}) - \frac{13}{5} R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{2}{5} R_1(\text{new})$

Iteration-3		C_j	600	400	0	0	0	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	A_2	**MinRatio** $\frac{X_B}{S_1}$
x_1	600	$\frac{68}{9}$	1	0	$-\frac{1}{9}$	0	$\frac{1}{3}$	0	---
A_2	M	$\frac{104}{9}$	0	0	$\left(\frac{13}{45}\right)$	-1	$-\frac{2}{3}$	1	$\frac{\frac{104}{9}}{\frac{13}{45}} = 40 \rightarrow$
x_2	400	$\frac{52}{9}$	0	1	$\frac{2}{45}$	0	$-\frac{1}{3}$	0	$\frac{\frac{52}{9}}{\frac{2}{45}} = 130$
$Z = \frac{61600}{9}$		Z_j	600	400	$\frac{13M}{45} - \frac{440}{9}$	$-M$	$-\frac{2M}{3} + \frac{200}{3}$	M	
		$C_j - Z_j$	0	0	$-\frac{13M}{45} + \frac{440}{9} \uparrow$	M	$\frac{2M}{3} - \frac{200}{3}$	0	

Negative minimum $C_j - Z_j$ is $-\frac{13M}{45} + \frac{440}{9}$ and its column index is 3. So, the entering variable is S_1.

Minimum ratio is 40 and its row index is 2. So, the leaving basis variable is A_2.

∴ The pivot element is $\frac{13}{45}$.

Entering $= S_1$, Departing $= A_2$, Key Element $= \frac{13}{45}$

$R_2(\text{new}) = R_2(\text{old}) \times \frac{45}{13}$

$R_1(\text{new}) = R_1(\text{old}) + \frac{1}{9} R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{2}{45} R_2(\text{new})$

Iteration-4		C_j	600	400	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio**
x_1	600	12	1	0	0	$-\frac{5}{13}$	$\frac{1}{13}$	
S_1	0	40	0	0	1	$-\frac{45}{13}$	$-\frac{30}{13}$	
x_2	400	4	0	1	0	$\frac{2}{13}$	$-\frac{3}{13}$	
$Z = 8800$		Z_j	**600**	**400**	**0**	$-\frac{2200}{13}$	$-\frac{600}{13}$	
		$C_j - Z_j$	0	0	0	$\frac{2200}{13}$	$\frac{600}{13}$	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 12, x_2 = 4$

Min $Z = 8800$

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Find solution using Simplex(BigM) method
MIN Z = 600x1 + 500x2
subject to
2x1 + x2 >= 80
x1 + 2x2 >= 60
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 600x_1 + 500x_2$$

subject to

$$2x_1 + x_2 \geq 80$$
$$x_1 + 2x_2 \geq 60$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\geq$ ' we should subtract surplus variable S_1 and add artificial variable A_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_2

After introducing surplus,artificial variables

$$\text{Min } Z = 600x_1 + 500x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$$

subject to

$$2x_1 + x_2 - S_1 + A_1 = 80$$
$$x_1 + 2x_2 - S_2 + A_2 = 60$$

and $x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Iteration-1		C_j	600	500	0	0	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**A_1**	**A_2**	**MinRatio** $\frac{X_B}{x_2}$
A_1	M	80	2	1	-1	0	1	0	$\frac{80}{1} = 80$
A_2	M	60	1	**(2)**	0	-1	0	1	$\frac{60}{2} = 30 \rightarrow$
$Z = 0$		Z_j	**$3M$**	**$3M$**	**$-M$**	**$-M$**	**M**	**M**	
		$C_j - Z_j$	$-3M + 600$	$-3M + 500 \uparrow$	M	M	0	0	

Negative minimum $C_j - Z_j$ is $-3M + 500$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 30 and its row index is 2. So, the leaving basis variable is A_2.

∴ The pivot element is 2.

Entering $= x_2$, Departing $= A_2$, Key Element $= 2$

$R_2(\text{new}) = R_2(\text{old}) \div 2$

$R_1(\text{new}) = R_1(\text{old}) - R_2(\text{new})$

Iteration-2		C_j	600	500	0	0	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	MinRatio $\frac{X_B}{x_1}$
A_1	M	50	$\left(\frac{3}{2}\right)$	0	-1	$\frac{1}{2}$	1	$\frac{50}{\frac{3}{2}} = \frac{100}{3} \rightarrow$
x_2	500	30	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$\frac{30}{\frac{1}{2}} = 60$
$Z = 15000$		Z_j	$\frac{3M}{2} + 250$	500	$-M$	$\frac{M}{2} - 250$	M	
		$C_j - Z_j$	$-\frac{3M}{2} + 350 \uparrow$	0	M	$-\frac{M}{2} + 250$	0	

Negative minimum $C_j - Z_j$ is $-\frac{3M}{2} + 350$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is $\frac{100}{3}$ and its row index is 1. So, the leaving basis variable is A_1.

∴ The pivot element is $\frac{3}{2}$.

Entering $= x_1$, Departing $= A_1$, Key Element $= \frac{3}{2}$

$R_1(\text{new}) = R_1(\text{old}) \times \frac{2}{3}$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{2} R_1(\text{new})$

Iteration-3		C_j	600	500	0	0	

B	C_B	X_B	x_1	x_2	S_1	S_2	MinRatio
x_1	600	$\frac{100}{3}$	1	0	$-\frac{2}{3}$	$\frac{1}{3}$	
x_2	500	$\frac{40}{3}$	0	1	$\frac{1}{3}$	$-\frac{2}{3}$	
$Z = \frac{80000}{3}$		Z_j	**600**	**500**	$-\frac{700}{3}$	$-\frac{400}{3}$	
		$C_j - Z_j$	0	0	$\frac{700}{3}$	$\frac{400}{3}$	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = \frac{100}{3}, x_2 = \frac{40}{3}$

Min $Z = \frac{80000}{3}$

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Find solution using Simplex(BigM) method
MIN Z = x1 + x2
subject to
2x1 + x2 >= 4
x1 + 7x2 >= 7
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = x_1 + x_2$$

subject to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\geq$ ' we should subtract surplus variable S_1 and add artificial variable A_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_2

After introducing surplus,artificial variables

$$\text{Min } Z = x_1 + x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$$

subject to

$$2x_1 + x_2 - S_1 + A_1 = 4$$
$$x_1 + 7x_2 - S_2 + A_2 = 7$$

and $x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Iteration-1		C_j	1	1	0	0	M	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	**MinRatio** $\frac{X_B}{x_2}$
A_1	M	4	2	1	-1	0	1	0	$\frac{4}{1} = 4$
A_2	M	7	1	(7)	0	-1	0	1	$\frac{7}{7} = 1 \rightarrow$
$Z = 0$		Z_j	$3M$	$8M$	$-M$	$-M$	M	M	
		$C_j - Z_j$	$-3M + 1$	$-8M + 1 \uparrow$	M	M	0	0	

Negative minimum $C_j - Z_j$ is $-8M + 1$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 1 and its row index is 2. So, the leaving basis variable is A_2.

∴ The pivot element is 7.

Entering $= x_2$, Departing $= A_2$, Key Element $= 7$

$R_2(\text{new}) = R_2(\text{old}) \div 7$

$R_1(\text{new}) = R_1(\text{old}) - R_2(\text{new})$

Iteration-2		C_j	1	1	0	0	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**A_1**	**MinRatio** $\dfrac{X_B}{x_1}$
A_1	M	3	$\left(\frac{13}{7}\right)$	0	-1	$\frac{1}{7}$	1	$\dfrac{3}{\frac{13}{7}} = \dfrac{21}{13}$ →
x_2	1	1	$\frac{1}{7}$	1	0	$-\frac{1}{7}$	0	$\dfrac{1}{\frac{1}{7}} = 7$
$Z = 1$		Z_j	$\frac{13M}{7} + \frac{1}{7}$	**1**	$-M$	$\frac{M}{7} - \frac{1}{7}$	M	
		$C_j - Z_j$	$-\frac{13M}{7} + \frac{6}{7}$ ↑	0	M	$-\frac{M}{7} + \frac{1}{7}$	0	

Negative minimum $C_j - Z_j$ is $-\frac{13M}{7} + \frac{6}{7}$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is $\frac{21}{13}$ and its row index is 1. So, the leaving basis variable is A_1.

∴ The pivot element is $\frac{13}{7}$.

Entering $= x_1$, Departing $= A_1$, Key Element $= \frac{13}{7}$

$R_1(\text{new}) = R_1(\text{old}) \times \frac{7}{13}$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{7}R_1(\text{new})$

Iteration-3		C_j	1	1	0	0	

B	C_B	X_B	x_1	x_2	S_1	S_2	MinRatio
x_1	1	$\frac{21}{13}$	1	0	$-\frac{7}{13}$	$\frac{1}{13}$	
x_2	1	$\frac{10}{13}$	0	1	$\frac{1}{13}$	$-\frac{2}{13}$	
$Z = \frac{31}{13}$		Z_j	**1**	**1**	$-\frac{6}{13}$	$-\frac{1}{13}$	
		$C_j - Z_j$	0	0	$\frac{6}{13}$	$\frac{1}{13}$	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = \frac{21}{13}, x_2 = \frac{10}{13}$

Min $Z = \frac{31}{13}$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 2x2
subject to
5x1 + x2 >= 10
2x1 + 2x2 >= 12
x1 + 4x2 >= 12
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to

$$5x_1 + x_2 \geq 10$$
$$2x_1 + 2x_2 \geq 12$$
$$x_1 + 4x_2 \geq 12$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\geq$ ' we should subtract surplus variable S_1 and add artificial variable A_1

2. As the constraint 2 is of type ' $\geq$ ' we should subtract surplus variable S_2 and add artificial variable A_2

3. As the constraint 3 is of type ' $\geq$ ' we should subtract surplus variable S_3 and add artificial variable A_3

After introducing surplus,artificial variables

$$\text{Max } Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_2 - MA_3$$

subject to

$$5x_1 + x_2 - S_1 + A_1 = 10$$
$$2x_1 + 2x_2 - S_2 + A_2 = 12$$
$$x_1 + 4x_2 - S_3 + A_3 = 12$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2, A_3 \geq 0$

Iteration-1		C_j	3	2	0	0	0	$-M$	$-M$	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**A_3**	**MinRatio** $\frac{X_B}{x_1}$
A_1	$-M$	10	(5)	1	-1	0	0	1	0	0	$\frac{10}{5} = 2 \rightarrow$
A_2	$-M$	12	2	2	0	-1	0	0	1	0	$\frac{12}{2} = 6$

A_3	$-M$	12	1	4	0	0	-1	0	0	1	$\frac{12}{1} = 12$
$Z = 0$		Z_j	$-8M$	$-7M$	M	M	M	$-M$	$-M$	$-M$	
		$C_j - Z_j$	$8M + 3$ ↑	$7M + 2$	$-M$	$-M$	$-M$	0	0	0	

Positive maximum $C_j - Z_j$ is $8M + 3$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 2 and its row index is 1. So, the leaving basis variable is A_1.

∴ The pivot element is 5.

Entering $= x_1$, Departing $= A_1$, Key Element $= 5$

$R_1(\text{new}) = R_1(\text{old}) \div 5$

$R_2(\text{new}) = R_2(\text{old}) - 2R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - R_1(\text{new})$

Iteration-2		C_j	3	2	0	0	0	$-M$	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_2**	**A_3**	**MinRatio** $\frac{X_B}{x_2}$
x_1	3	2	1	0.2	-0.2	0	0	0	0	$\frac{2}{0.2} = 10$
A_2	$-M$	8	0	1.6	0.4	-1	0	1	0	$\frac{8}{1.6} = 5$
A_3	$-M$	10	0	**(3.8)**	0.2	0	-1	0	1	$\frac{10}{3.8} = 2.6316$ →
$Z = 6$		Z_j	3	$-\frac{27M}{5} + 0.6$	$-\frac{3M}{5} - 0.6$	M	M	$-M$	$-M$	
		$C_j - Z_j$	0	$\frac{27M}{5} + 1.4$ ↑	$\frac{3M}{5} + 0.6$	$-M$	$-M$	0	0	

Positive maximum $C_j - Z_j$ is $\frac{27M}{5} + 1.4$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 2.6316 and its row index is 3. So, the leaving basis variable is A_3.

∴ The pivot element is 3.8.

Entering $= x_2$, Departing $= A_3$, Key Element $= 3.8$

$R_3(\text{new}) = R_3(\text{old}) \times 0.2632$

$R_1(\text{new}) = R_1(\text{old}) - 0.2R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - 1.6R_3(\text{new})$

Iteration-3		C_j	3	2	0	0	0	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_2**	**MinRatio** $\frac{X_B}{S_3}$
x_1	3	1.4737	1	0	-0.2105	0	0.0526	0	$\frac{1.4737}{0.0526} = 28$
A_2	$-M$	3.7895	0	0	0.3158	-1	**(0.4211)**	1	$\frac{3.7895}{0.4211} = 9 \rightarrow$
x_2	2	2.6316	0	1	0.0526	0	-0.2632	0	---
$Z = 9.6842$		**Z_j**	**3**	**2**	$-\frac{6M}{19} - 0.5263$	**M**	$-\frac{8M}{19} - 0.3684$	**$-M$**	
		$C_j - Z_j$	0	0	$\frac{6M}{19} + 0.5263$	$-M$	$\frac{8M}{19} + 0.3684 \uparrow$	0	

Positive maximum $C_j - Z_j$ is $\frac{8M}{19} + 0.3684$ and its column index is 5. So, the entering variable is S_3.

Minimum ratio is 9 and its row index is 2. So, the leaving basis variable is A_2.

∴ The pivot element is 0.4211.

Entering $= S_3$, Departing $= A_2$, Key Element $= 0.4211$

$R_2(\text{new}) = R_2(\text{old}) \times 2.375$

$R_1(\text{new}) = R_1(\text{old}) - 0.0526R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + 0.2632R_2(\text{new})$

Iteration-4		C_j	3	2	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**MinRatio** $\frac{X_B}{S_2}$
x_1	3	1	1	0	-0.25	**(0.125)**	0	

								$\frac{1}{0.125} = 8 \rightarrow$
S_3	0	9	0	0	0.75	-2.375	1	---
x_2	2	5	0	1	0.25	-0.625	0	---
$Z = 13$		Z_j	**3**	**2**	**-0.25**	**-0.875**	**0**	
		$C_j - Z_j$	0	0	0.25	0.875 $\uparrow$	0	

Positive maximum $C_j - Z_j$ is 0.875 and its column index is 4. So, the entering variable is S_2.

Minimum ratio is 8 and its row index is 1. So, the leaving basis variable is x_1.

∴ The pivot element is 0.125.

Entering $= S_2$, Departing $= x_1$, Key Element $= 0.125$

$R_1(\text{new}) = R_1(\text{old}) \times 8$

$R_2(\text{new}) = R_2(\text{old}) + 2.375 R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + 0.625 R_1(\text{new})$

Iteration-5		C_j	3	2	0	0	0	
B	***C_B***	***X_B***	***x_1***	***x_2***	***S_1***	***S_2***	***S_3***	**MinRatio** $\frac{X_B}{S_1}$
S_2	0	8	8	0	**(-2)**	1	0	---
S_3	0	28	19	0	-4	0	1	---
x_2	2	10	5	1	-1	0	0	---
$Z = 20$		Z_j	**10**	**2**	**-2**	**0**	**0**	
		$C_j - Z_j$	-7	0	2 $\uparrow$	0	0	

Variable S_1 should enter into the basis, but all the coefficients in the S_1 column are negative or zero. So S_1 can not be entered into the basis.

Hence, the solution to the given problem is unbounded.

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Find solution using Simplex(BigM) method
MAX Z = 2x1 + 4x2
subject to
5x1 + 4x2 <= 200
3x1 + 5x2 <= 150
5x1 + 4x2 >= 100
8x1 + 4x2 >= 80
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 2x_1 + 4x_2$$

subject to

$$5x_1 + 4x_2 \le 200$$
$$3x_1 + 5x_2 \le 150$$
$$5x_1 + 4x_2 \ge 100$$
$$8x_1 + 4x_2 \ge 80$$

and $x_1, x_2 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\ge$ ' we should subtract surplus variable S_3 and add artificial variable A_1

4. As the constraint 4 is of type ' $\ge$ ' we should subtract surplus variable S_4 and add artificial variable A_2

After introducing slack,surplus,artificial variables

$$\text{Max } Z = 2x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4 - MA_1 - MA_2$$

subject to

$$5x_1 + 4x_2 + S_1 = 200$$
$$3x_1 + 5x_2 + S_2 = 150$$
$$5x_1 + 4x_2 - S_3 + A_1 = 100$$
$$8x_1 + 4x_2 - S_4 + A_2 = 80$$

and $x_1, x_2, S_1, S_2, S_3, S_4, A_1, A_2 \ge 0$

Iteration-1		C_j	2	4	0	0	0	0	$-M$	$-M$	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	A_1	A_2	MinRatio $\frac{X_B}{x_1}$

S_1	0	200	5	4	1	0	0	0	0	0	$\frac{200}{5} = 40$
S_2	0	150	3	5	0	1	0	0	0	0	$\frac{150}{3} = 50$
A_1	$-M$	100	5	4	0	0	-1	0	1	0	$\frac{100}{5} = 20$
A_2	$-M$	80	**(8)**	4	0	0	0	-1	0	1	$\frac{80}{8} = 10 \rightarrow$
$Z = 0$		Z_j	**$-13M$**	**$-8M$**	**0**	**0**	**M**	**M**	**$-M$**	**$-M$**	
		$C_j - Z_j$	$13M + 2 \uparrow$	$8M + 4$	0	0	$-M$	$-M$	0	0	

Positive maximum $C_j - Z_j$ is $13M + 2$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 10 and its row index is 4. So, the leaving basis variable is A_2.

$\therefore$ The pivot element is 8.

Entering $= x_1$, Departing $= A_2$, Key Element $= 8$

$R_4(\text{new}) = R_4(\text{old}) \div 8$

$R_1(\text{new}) = R_1(\text{old}) - 5R_4(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - 3R_4(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - 5R_4(\text{new})$

Iteration-2		C_j	2	4	0	0	0	0	$-M$	
B	***C_B***	***X_B***	***x_1***	***x_2***	***S_1***	***S_2***	***S_3***	***S_4***	***A_1***	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	150	0	$\frac{3}{2}$	1	0	0	$\frac{5}{8}$	0	$\frac{150}{\frac{3}{2}} = 100$
S_2	0	120	0	$\frac{7}{2}$	0	1	0	$\frac{3}{8}$	0	$\frac{120}{\frac{7}{2}} = \frac{240}{7}$
A_1	$-M$	50	0	$\frac{3}{2}$	0	0	-1	$\frac{5}{8}$	1	$\frac{50}{\frac{3}{2}} = \frac{100}{3}$
	2	10	1		0	0	0		0	

x_1				$\left(\frac{1}{2}\right)$				$-\frac{1}{8}$		$\frac{10}{\frac{1}{2}} = 20 \rightarrow$
$Z = 20$		Z_j	2	$-\frac{3M}{2} + 1$	0	0	M	$-\frac{5M}{8} - \frac{1}{4}$	$-M$	
		$C_j - Z_j$	0	$\frac{3M}{2} + 3 \uparrow$	0	0	$-M$	$\frac{5M}{8} + \frac{1}{4}$	0	

Positive maximum $C_j - Z_j$ is $\frac{3M}{2} + 3$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 20 and its row index is 4. So, the leaving basis variable is x_1.

$\therefore$ The pivot element is $\frac{1}{2}$.

Entering $= x_2$, Departing $= x_1$, Key Element $= \frac{1}{2}$

$R_4(\text{new}) = R_4(\text{old}) \times 2$

$R_1(\text{new}) = R_1(\text{old}) - \frac{3}{2} R_4(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - \frac{7}{2} R_4(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{3}{2} R_4(\text{new})$

Iteration-3		C_j	2	4	0	0	0	0	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**S_4**	**A_1**	**MinRatio** $\frac{X_B}{S_4}$
S_1	0	120	-3	0	1	0	0	1	0	$\frac{120}{1} = 120$
S_2	0	50	-7	0	0	1	0	$\frac{5}{4}$	0	$\frac{50}{\frac{5}{4}} = 40$
A_1	$-M$	20	-3	0	0	0	-1	**(1)**	1	$\frac{20}{1} = 20 \rightarrow$
x_2	4	20	2	1	0	0	0	$-\frac{1}{4}$	0	---

$Z = 80$		**Z_j**	**$3M + 8$**	**4**	**0**	**0**	**M**	**$-M - 1$**	**$-M$**	
		$C_j - Z_j$	$-3M - 6$	0	0	0	$-M$	$M + 1$ ↑	0	

Positive maximum $C_j - Z_j$ is $M + 1$ and its column index is 6. So, the entering variable is S_4.

Minimum ratio is 20 and its row index is 3. So, the leaving basis variable is A_1.

∴ The pivot element is 1.

Entering $= S_4$, Departing $= A_1$, Key Element $= 1$

$R_3(\text{new}) = R_3(\text{old})$

$R_1(\text{new}) = R_1(\text{old}) - R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - \frac{5}{4} R_3(\text{new})$

$R_4(\text{new}) = R_4(\text{old}) + \frac{1}{4} R_3(\text{new})$

Iteration-4		C_j	2	4	0	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**S_4**	**MinRatio** $\frac{X_B}{S_3}$
S_1	0	100	0	0	1	0	1	0	$\frac{100}{1} = 100$
S_2	0	25	$-\frac{13}{4}$	0	0	1	$\left(\frac{5}{4}\right)$	0	$\frac{25}{\frac{5}{4}} = 20$ →
S_4	0	20	-3	0	0	0	-1	1	---
x_2	4	25	$\frac{5}{4}$	1	0	0	$-\frac{1}{4}$	0	---
$Z = 100$		**Z_j**	**5**	**4**	**0**	**0**	**-1**	**0**	
		$C_j - Z_j$	-3	0	0	0	1 ↑	0	

Positive maximum $C_j - Z_j$ is 1 and its column index is 5. So, the entering variable is S_3.

Minimum ratio is 20 and its row index is 2. So, the leaving basis variable is S_2.

∴ The pivot element is $\frac{5}{4}$.

Entering $= S_3$, Departing $= S_2$, Key Element $= \frac{5}{4}$

$R_2(\text{new}) = R_2(\text{old}) \times \frac{4}{5}$

$R_1(\text{new}) = R_1(\text{old}) - R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + R_2(\text{new})$

$R_4(\text{new}) = R_4(\text{old}) + \frac{1}{4} R_2(\text{new})$

Iteration-5		C_j	2	4	0	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	**MinRatio**
S_1	0	80	$\frac{13}{5}$	0	1	$-\frac{4}{5}$	0	0	
S_3	0	20	$-\frac{13}{5}$	0	0	$\frac{4}{5}$	1	0	
S_4	0	40	$-\frac{28}{5}$	0	0	$\frac{4}{5}$	0	1	
x_2	4	30	$\frac{3}{5}$	1	0	$\frac{1}{5}$	0	0	
$Z = 120$		Z_j	$\frac{12}{5}$	**4**	**0**	$\frac{4}{5}$	**0**	**0**	
		$C_j - Z_j$	$-\frac{2}{5}$	0	0	$-\frac{4}{5}$	0	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 30$

Max $Z = 120$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 2x2 + 3x3 - x4
subject to
x1 + 2x2 + 3x3 = 15
2x1 + x2 + 5x3 = 20
x1 + 2x2 + x3 + x4 = 10
and x1,x2,x3,x4 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 2x_2 + 3x_3 - x_4$$

subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$

and $x_1, x_2, x_3, x_4 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' = ' we should add artificial variable A_1

2. As the constraint 2 is of type ' = ' we should add artificial variable A_2

3. As the constraint 3 is of type ' = ' we should add artificial variable A_3

After introducing artificial variables

$$\text{Max } Z = 3x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$$

subject to

$$x_1 + 2x_2 + 3x_3 + A_1 = 15$$
$$2x_1 + x_2 + 5x_3 + A_2 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$$

and $x_1, x_2, x_3, x_4, A_1, A_2, A_3 \geq 0$

Iteration-1		C_j	3	2	3	-1	$-M$	$-M$	$-M$	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**A_1**	**A_2**	**A_3**	**MinRatio** $\frac{X_B}{x_3}$
A_1	$-M$	15	1	2	3	0	1	0	0	$\frac{15}{3} = 5$
A_2	$-M$	20	2	1	**(5)**	0	0	1	0	$\frac{20}{5} = 4 \rightarrow$

A_3	$-M$	10	1	2	1	1	0	0	1	$\frac{10}{1} = 10$
$Z = 0$		Z_j	$-4M$	$-5M$	$-9M$	$-M$	$-M$	$-M$	$-M$	
		$C_j - Z_j$	$4M + 3$	$5M + 2$	$9M + 3$ ↑	$M - 1$	0	0	0	

Positive maximum $C_j - Z_j$ is $9M + 3$ and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 4 and its row index is 2. So, the leaving basis variable is A_2.

∴ The pivot element is 5.

Entering $= x_3$, Departing $= A_2$, Key Element $= 5$

$R_2(\text{new}) = R_2(\text{old}) \div 5$

$R_1(\text{new}) = R_1(\text{old}) - 3R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - R_2(\text{new})$

Iteration-2		C_j	3	2	3	-1	$-M$	$-M$	
B	C_B	X_B	x_1	x_2	x_3	x_4	A_1	A_3	**MinRatio** $\frac{X_B}{x_2}$
A_1	$-M$	3	$-\frac{1}{5}$	$\left(\frac{7}{5}\right)$	0	0	1	0	$\frac{3}{\frac{7}{5}} = \frac{15}{7}$ →
x_3	3	4	$\frac{2}{5}$	$\frac{1}{5}$	1	0	0	0	$\frac{4}{\frac{1}{5}} = 20$
A_3	$-M$	6	$\frac{3}{5}$	$\frac{9}{5}$	0	1	0	1	$\frac{6}{\frac{9}{5}} = \frac{10}{3}$
$Z = 12$		Z_j	$-\frac{2M}{5} + \frac{6}{5}$	$-\frac{16M}{5} + \frac{3}{5}$	3	$-M$	$-M$	$-M$	
		$C_j - Z_j$	$\frac{2M}{5} + \frac{9}{5}$	$\frac{16M}{5} + \frac{7}{5}$ ↑	0	$M - 1$	0	0	

Positive maximum $C_j - Z_j$ is $\frac{16M}{5} + \frac{7}{5}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{15}{7}$ and its row index is 1. So, the leaving basis variable is A_1.

$\therefore$ The pivot element is $\frac{7}{5}$.

Entering $= x_2$, Departing $= A_1$, Key Element $= \frac{7}{5}$

$R_1(\text{new}) = R_1(\text{old}) \times \frac{5}{7}$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{5} R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{9}{5} R_1(\text{new})$

Iteration-3		C_j	3	2	3	-1	-M	
B	C_B	X_B	x_1	x_2	x_3	x_4	A_3	**MinRatio** $\frac{X_B}{x_4}$
x_2	2	$\frac{15}{7}$	$-\frac{1}{7}$	1	0	0	0	---
x_3	3	$\frac{25}{7}$	$\frac{3}{7}$	0	1	0	0	---
A_3	$-M$	$\frac{15}{7}$	$\frac{6}{7}$	0	0	**(1)**	1	$\frac{\frac{15}{7}}{1} = \frac{15}{7} \rightarrow$
Z = 15		Z_j	$-\frac{6M}{7}+1$	2	3	$-M$	$-M$	
		$C_j - Z_j$	$\frac{6M}{7}+2$	0	0	$M-1 \uparrow$	0	

Positive maximum $C_j - Z_j$ is $M - 1$ and its column index is 4. So, the entering variable is x_4.

Minimum ratio is $\frac{15}{7}$ and its row index is 3. So, the leaving basis variable is A_3.

$\therefore$ The pivot element is 1.

Entering $= x_4$, Departing $= A_3$, Key Element $= 1$

$R_3(\text{new}) = R_3(\text{old})$

R_1(new) = R_1(old)

R_2(new) = R_2(old)

Iteration-4		C_j	3	2	3	-1	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**MinRatio** $\frac{X_B}{x_1}$
x_2	2	$\frac{15}{7}$	$-\frac{1}{7}$	1	0	0	---
x_3	3	$\frac{25}{7}$	$\frac{3}{7}$	0	1	0	$\frac{\frac{25}{7}}{\frac{3}{7}} = \frac{25}{3}$
x_4	-1	$\frac{15}{7}$	$\left(\frac{6}{7}\right)$	0	0	1	$\frac{\frac{15}{7}}{\frac{6}{7}} = \frac{5}{2}$ →
$Z = \frac{90}{7}$		Z_j	$\frac{1}{7}$	2	3	-1	
		$C_j - Z_j$	$\frac{20}{7}$ ↑	0	0	0	

Positive maximum $C_j - Z_j$ is $\frac{20}{7}$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is $\frac{5}{2}$ and its row index is 3. So, the leaving basis variable is x_4.

∴ The pivot element is $\frac{6}{7}$.

Entering $= x_1$, Departing $= x_4$, Key Element $= \frac{6}{7}$

R_3(new) = R_3(old) $\times \frac{7}{6}$

R_1(new) = R_1(old)+ $\frac{1}{7} R_3$(new)

R_2(new) = R_2(old)- $\frac{3}{7} R_3$(new)

Iteration-5			3	2	3	-1	

		C_j					
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**MinRatio**
x_2	2	$\frac{5}{2}$	0	1	0	$\frac{1}{6}$	
x_3	3	$\frac{5}{2}$	0	0	1	$-\frac{1}{2}$	
x_1	3	$\frac{5}{2}$	1	0	0	$\frac{7}{6}$	
$Z = 20$		**Z_j**	**3**	**2**	**3**	**$\frac{7}{3}$**	
		$C_j - Z_j$	0	0	0	$-\frac{10}{3}$	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = \frac{5}{2}, x_2 = \frac{5}{2}, x_3 = \frac{5}{2}, x_4 = 0$

Max $Z = 20$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 7x2 + 6x3
subject to
2x1 + 4x2 + 7x3 >= 4
x1 + 7x2 + 2x3 <= 7
3x1 + 6x2 + 5x3 <= 25
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 7x_2 + 6x_3$$

subject to

$$2x_1 + 4x_2 + 7x_3 \geq 4$$
$$x_1 + 7x_2 + 2x_3 \leq 7$$
$$3x_1 + 6x_2 + 5x_3 \leq 25$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\geq$ ' we should subtract surplus variable S_1 and add artificial variable A_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_3

After introducing slack,surplus,artificial variables

$$\text{Max } Z = 3x_1 + 7x_2 + 6x_3 + 0S_1 + 0S_2 + 0S_3 - MA_1$$

subject to

$$2x_1 + 4x_2 + 7x_3 - S_1 + A_1 = 4$$
$$x_1 + 7x_2 + 2x_3 + S_2 = 7$$
$$3x_1 + 6x_2 + 5x_3 + S_3 = 25$$

and $x_1, x_2, x_3, S_1, S_2, S_3, A_1 \geq 0$

Iteration-1		C_j	3	7	6	0	0	0	$-M$	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	A_1	**MinRatio** $\frac{X_B}{x_3}$
A_1	$-M$	4	2	4	(7)	-1	0	0	1	$\frac{4}{7} = \frac{4}{7} \rightarrow$
S_1	0	7	1	7	2	0	1	0	0	$\frac{7}{2} = \frac{7}{2}$

S_2	0	25	3	6	5	0	0	1	0	$\frac{25}{5} = 5$
$Z = 0$		Z_j	$-2M$	$-4M$	$-7M$	M	0	0	$-M$	
		$C_j - Z_j$	$2M + 3$	$4M + 7$	$7M + 6 \uparrow$	$-M$	0	0	0	

Positive maximum $C_j - Z_j$ is $7M + 6$ and its column index is 3. So, the entering variable is x_3.

Minimum ratio is $\frac{4}{7}$ and its row index is 1. So, the leaving basis variable is A_1.

$\therefore$ The pivot element is 7.

Entering $= x_3$, Departing $= A_1$, Key Element $= 7$

$R_1(\text{new}) = R_1(\text{old}) \div 7$

$R_2(\text{new}) = R_2(\text{old}) - 2R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - 5R_1(\text{new})$

Iteration-2		C_j	3	7	6	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**	**MinRatio** $\frac{X_B}{x_2}$
x_3	6	$\frac{4}{7}$	$\frac{2}{7}$	$\frac{4}{7}$	1	$-\frac{1}{7}$	0	0	$\frac{\frac{4}{7}}{\frac{4}{7}} = 1$
S_1	0	$\frac{41}{7}$	$\frac{3}{7}$	$\left(\frac{41}{7}\right)$	0	$\frac{2}{7}$	1	0	$\frac{\frac{41}{7}}{\frac{41}{7}} = 1 \rightarrow$
S_2	0	$\frac{155}{7}$	$\frac{11}{7}$	$\frac{22}{7}$	0	$\frac{5}{7}$	0	1	$\frac{\frac{155}{7}}{\frac{22}{7}} = \frac{155}{22}$
$Z = \frac{24}{7}$		Z_j	$\frac{12}{7}$	$\frac{24}{7}$	6	$-\frac{6}{7}$	0	0	
		$C_j - Z_j$	$\frac{9}{7}$	$\frac{25}{7} \uparrow$	0	$\frac{6}{7}$	0	0	

Positive maximum C_j - Z_j is $\frac{25}{7}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 1 and its row index is 2. So, the leaving basis variable is S_1.

∴ The pivot element is $\frac{41}{7}$.

Entering $= x_2$, Departing $= S_1$, Key Element $= \frac{41}{7}$

$R_2(\text{new}) = R_2(\text{old}) \times \frac{7}{41}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{4}{7} R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{22}{7} R_2(\text{new})$

Iteration-3		C_j	3	7	6	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**	**MinRatio** $\frac{X_B}{x_1}$
x_3	6	0	$\left(\frac{10}{41}\right)$	0	1	$-\frac{7}{41}$	$-\frac{4}{41}$	0	$\frac{0}{\frac{10}{41}} = 0$ →
x_2	7	1	$\frac{3}{41}$	1	0	$\frac{2}{41}$	$\frac{7}{41}$	0	$\frac{1}{\frac{3}{41}} = \frac{41}{3}$
S_2	0	19	$\frac{55}{41}$	0	0	$\frac{23}{41}$	$-\frac{22}{41}$	1	$\frac{19}{\frac{55}{41}} = \frac{779}{55}$
$Z = 7$		**Z_j**	**$\frac{81}{41}$**	**7**	**6**	**$-\frac{28}{41}$**	**$\frac{25}{41}$**	**0**	
		$C_j - Z_j$	$\frac{42}{41}$ ↑	0	0	$\frac{28}{41}$	$-\frac{25}{41}$	0	

Positive maximum C_j - Z_j is $\frac{42}{41}$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 0 and its row index is 1. So, the leaving basis variable is x_3.

∴ The pivot element is $\frac{10}{41}$.

Entering $= x_1$, Departing $= x_3$, Key Element $= \frac{10}{41}$

$R_1(\text{new}) = R_1(\text{old}) \times \frac{41}{10}$

$R_2(\text{new}) = R_2(\text{old}) - \frac{3}{41} R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{55}{41} R_1(\text{new})$

Iteration-4		C_j	3	7	6	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{S_1}$
x_1	3	0	1	0	$\frac{41}{10}$	$-\frac{7}{10}$	$-\frac{2}{5}$	0	---
x_2	7	1	0	1	$-\frac{3}{10}$	$\left(\frac{1}{10}\right)$	$\frac{1}{5}$	0	$\frac{1}{\frac{1}{10}} = 10 \rightarrow$
S_2	0	19	0	0	$-\frac{11}{2}$	$\frac{3}{2}$	0	1	$\frac{19}{\frac{3}{2}} = \frac{38}{3}$
$Z = 7$		Z_j	3	7	$\frac{51}{5}$	$-\frac{7}{5}$	$\frac{1}{5}$	0	
		$C_j - Z_j$	0	0	$-\frac{21}{5}$	$\frac{7}{5} \uparrow$	$-\frac{1}{5}$	0	

Positive maximum $C_j - Z_j$ is $\frac{7}{5}$ and its column index is 4. So, the entering variable is S_1.

Minimum ratio is 10 and its row index is 2. So, the leaving basis variable is x_2.

$\therefore$ The pivot element is $\frac{1}{10}$.

Entering $= S_1$, Departing $= x_2$, Key Element $= \frac{1}{10}$

$R_2(\text{new}) = R_2(\text{old}) \times 10$

$R_1(\text{new}) = R_1(\text{old}) + \frac{7}{10} R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{3}{2} R_2(\text{new})$

Iteration-5		C_j	3	7	6	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	**MinRatio**
x_1	3	7	1	7	2	0	1	0	
S_1	0	10	0	10	-3	1	2	0	
S_2	0	4	0	-15	-1	0	-3	1	
$Z = 21$		Z_j	**3**	**21**	**6**	**0**	**3**	**0**	
		$C_j - Z_j$	0	-14	0	0	-3	0	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 7, x_2 = 0, x_3 = 0$

Max $Z = 21$

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Find solution using Simplex(BigM) method
MIN Z = 4x1 - 2x2
subject to
x1 + x2 <= 14
3x1 + 2x2 >= 36
2x1 + x2 >= 24
and x1,x2 >= 0

Solution:
Problem is

$$\text{Min } Z = 4x_1 - 2x_2$$

subject to

$$x_1 + x_2 \le 14$$
$$3x_1 + 2x_2 \ge 36$$
$$2x_1 + x_2 \ge 24$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\ge$ ' we should subtract surplus variable S_2 and add artificial variable A_1

3. As the constraint 3 is of type ' $\ge$ ' we should subtract surplus variable S_3 and add artificial variable A_2

After introducing slack,surplus,artificial variables

$$\text{Min } Z = 4x_1 - 2x_2 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2$$

subject to

$$x_1 + x_2 + S_1 = 14$$
$$3x_1 + 2x_2 - S_2 + A_1 = 36$$
$$2x_1 + x_2 - S_3 + A_2 = 24$$

and $x_1, x_2, S_1, S_2, S_3, A_1, A_2 \ge 0$

Iteration-1		C_j	4	-2	0	0	0	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	14	1	1	1	0	0	0	0	$\frac{14}{1} = 14$
A_1	M	36	3	2	0	-1	0	1	0	$\frac{36}{3} = 12$

A_2	M	24	(2)	1	0	0	-1	0	1	$\frac{24}{2} = 12 \rightarrow$
$Z = 0$		Z_j	$5M$	$3M$	0	$-M$	$-M$	M	M	
		$C_j - Z_j$	$-5M + 4 \uparrow$	$-3M - 2$	0	M	M	0	0	

Negative minimum $C_j - Z_j$ is $-5M + 4$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 12 and its row index is 3. So, the leaving basis variable is A_2.

∴ The pivot element is 2.

Entering $= x_1$, Departing $= A_2$, Key Element $= 2$

$R_3(\text{new}) = R_3(\text{old}) \div 2$

$R_1(\text{new}) = R_1(\text{old}) - R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - 3R_3(\text{new})$

Iteration-2		C_j	4	-2	0	0	0	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**A_1**	**MinRatio** $\frac{X_B}{S_3}$
S_1	0	2	0	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$\frac{2}{\frac{1}{2}} = 4$
A_1	M	0	0	$\frac{1}{2}$	0	-1	$\left(\frac{3}{2}\right)$	1	$\frac{0}{\frac{3}{2}} = 0 \rightarrow$
x_1	4	12	1	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	---
$Z = 48$		Z_j	4	$\frac{M}{2} + 2$	0	$-M$	$\frac{3M}{2} - 2$	M	
		$C_j - Z_j$	0	$-\frac{M}{2} - 4$	0	M	$-\frac{3M}{2} + 2 \uparrow$	0	

Negative minimum $C_j - Z_j$ is $-\frac{3M}{2} + 2$ and its column index is 5. So, the entering variable is S_3.

Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is A_1.

$\therefore$ The pivot element is $\frac{3}{2}$.

Entering $= S_3$, Departing $= A_1$, Key Element $= \frac{3}{2}$

$R_2(\text{new}) = R_2(\text{old}) \times \frac{2}{3}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{2} R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + \frac{1}{2} R_2(\text{new})$

Iteration-3		C_j	4	-2	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**MinRatio $\frac{X_B}{x_2}$**
S_1	0	2	0	$\frac{1}{3}$	1	$\frac{1}{3}$	0	$\frac{2}{\frac{1}{3}} = 6$
S_3	0	0	0	$\left(\frac{1}{3}\right)$	0	$-\frac{2}{3}$	1	$\frac{0}{\frac{1}{3}} = 0 \rightarrow$
x_1	4	12	1	$\frac{2}{3}$	0	$-\frac{1}{3}$	0	$\frac{12}{\frac{2}{3}} = 18$
$Z = 48$		Z_j	4	$\frac{8}{3}$	0	$-\frac{4}{3}$	0	
		$C_j - Z_j$	0	$-\frac{14}{3} \uparrow$	0	$\frac{4}{3}$	0	

Negative minimum $C_j - Z_j$ is $-\frac{14}{3}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is S_3.

$\therefore$ The pivot element is $\frac{1}{3}$.

Entering $= x_2$, Departing $= S_3$, Key Element $= \frac{1}{3}$

$R_2(\text{new}) = R_2(\text{old}) \times 3$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{3} R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{2}{3} R_2(\text{new})$

Iteration-4		C_j	4	-2	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{S_2}$
S_1	0	2	0	0	1	**(1)**	-1	$\frac{2}{1} = 2 \rightarrow$
x_2	-2	0	0	1	0	-2	3	---
x_1	4	12	1	0	0	1	-2	$\frac{12}{1} = 12$
$Z = 48$		Z_j	**4**	**-2**	**0**	**8**	**-14**	
		$C_j - Z_j$	0	0	0	-8 $\uparrow$	14	

Negative minimum $C_j - Z_j$ is -8 and its column index is 4. So, the entering variable is S_2.

Minimum ratio is 2 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 1.

Entering $= S_2$, Departing $= S_1$, Key Element $= 1$

$R_1(\text{new}) = R_1(\text{old})$

$R_2(\text{new}) = R_2(\text{old}) + 2R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - R_1(\text{new})$

Iteration-5		C_j	4	-2	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio**
S_2	0	2	0	0	1	1	-1	
x_2	-2	4	0	1	2	0	1	
x_1	4	10	1	0	-1	0	-1	
$Z = 32$		Z_j	**4**	**-2**	**-8**	**0**	**-6**	

		$C_j - Z_j$	0	0	8	0	6	

Since all $C_j - Z_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 10, x_2 = 4$

Min $Z = 32$

Solution is provided by AtoZmath.com