Guideline to Simplex Method

Step1. Check if the linear programming problem is a **standard maximization problem in standard form**, i.e., if all the following conditions are satisfied:

- It's to **maximize** an objective function;
- All variables should be non-negative (i.e. ≥ 0).
- Constraints should all be $\leq$ a non-negative.

Step 2. Create **slack variables** to convert the inequalities to equations.

Step 3. Write the *objective* function as an equation in the form "left hand side"= 0 where terms involving variables are negative. *Example*: $Z = 3x + 4y$ *becomes* $-3x - 4y + Z = 0$.

Step 4. Place the system of equations with slack variables, into a matrix. Place the revised objective equation in the bottom row.

Step 5. Select **pivot column** by finding the most negative indicator. (**Indicators** are those elements in bottom row except last two elements in that row)

Step 6. Select **pivot row**. (Divide the last column by pivot column for each corresponding entry except bottom entry and negative entries. Choose the smallest positive result. The corresponding row is the pivot row. In case there is no positive entry in pivot column above dashed line, there is no optimal solutions)

Step 7. Find **pivot**: Circle the pivot entry at the intersection of the pivot column and the pivot row, and identify entering variable and exit variable at mean time. Divide pivot by itself in that row to obtain 1. (NEVER SWAP TWO ROWS in Simplex Method!) Also obtain zeros for all rest entries in pivot column by row operations.

Step 8. Do we get all **nonnegative** indicators? If yes, we may stop. Otherwise repeat step 5 to step 7.

Step 9. Read the results: Correspond the last column entries to the variables in front of the first column. The variables not showing are automatically equal to 0.

Example. Maximize $P = 3x_1 + x_2$ Subject to: $\begin{aligned} 2x_1 + x_2 &\le 8 \\ 2x_1 + 3x_2 &\le 12 \\ x_1, x_2 &\ge 0 \end{aligned}$

<u>Solution</u>

Step 1. This is of course a standard maximization problem in standard form.

Step 2. Rewrite the two problem constraints as equations by using slack variables:

$$\begin{aligned} 2x_1 + x_2 + s_1 \quad\quad &= 8 \\ 2x_1 + 3x_2 \quad + s_2 &= 12 \end{aligned}$$

Step 3. Rewrite the objective function in the form $-3x_1 - x_2 + P = 0$. Put it together with the problem constraints: $\begin{aligned} 2x_1 + x_2 + s_1 \quad\quad\quad &= 8 \\ 2x_1 + 3x_2 \quad + s_2 \quad\; &= 12 \\ -3x_1 - x_2 \quad\quad\quad + \mathrm{P} &= 12 \end{aligned}$, we get a linear system with 5 variables and 3 equations, which is called **initial system**.

Step 4. Write the initial system in matrix form (**initial simplex tableau**). See below.

Step 5 to step 9:

$$\begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & P & \end{array} \\ \left[\begin{array}{ccccc|c} 2 \text{ pivot} & 1 & 1 & 0 & 0 & 8 \\ 2 & 3 & 0 & 1 & 0 & 12 \\ \hdashline -3 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \frac{8}{2} = 4 \text{ smallest positive pivot row} \\ \frac{12}{2} = 6 \\ \\ \end{array}$$

most negative indicator pivot column

$$\tfrac{1}{2}R_1 \to R_1 \quad \begin{array}{c} x_1 \\ s_2 \\ P \end{array} \left[\begin{array}{ccccc|c} 1 & 1/2 & 1/2 & 0 & 0 & 4 \\ 2 & 3 & 0 & 1 & 0 & 12 \\ \hdashline -3 & -1 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{c} -2R_1 + R_2 \to R_2 \\ 3R_1 + R_3 \to R_3 \end{array} \quad \begin{array}{c} x_1 \\ s_2 \\ P \end{array} \left[\begin{array}{ccccc|c} 1 & 1/2 & 1/2 & 0 & 0 & 4 \\ 0 & 2 & -1 & 1 & 0 & 4 \\ \hdashline \underbrace{0 \quad 1/2 \quad 3/2 \quad 0}_{\text{no negative indicators, stop!}} & & & & 1 & 12 \end{array}\right]$$

max. value of P
@ $x_1 = 4$, $x_2 = 0$

Check: compare to the method we did in 5-3, we got same answer!

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Find solution using Simplex(BigM) method
MAX Z = 2x1 + 3x2
subject to
6x1 + 4x2 <= 4
2x1 + 4x2 <= 6
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 2x_1 + 3x_2$$

subject to

$$6x_1 + 4x_2 \le 4$$
$$2x_1 + 4x_2 \le 6$$

and $x_1, x_2 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 2x_1 + 3x_2 + 0S_1 + 0S_2$$

subject to

$$6x_1 + 4x_2 + S_1 = 4$$
$$2x_1 + 4x_2 + S_2 = 6$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	2	3	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	4	6	**(4)**	1	0	$\frac{4}{4} = 1 \rightarrow$
S_2	0	6	2	4	0	1	$\frac{6}{4} = \frac{3}{2}$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	2	3 $\uparrow$	0	0	

Positive maximum $C_j - Z_j$ is 3 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 1 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 4.

Entering $= x_2$, Departing $= S_1$, Key Element $= 4$

$R_1(\text{new}) = R_1(\text{old}) \div 4$

$R_2(\text{new}) = R_2(\text{old}) - 4R_1(\text{new})$

Iteration-2		C_j	2	3	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio**
x_2	3	1	$\frac{3}{2}$	1	$\frac{1}{4}$	0	
S_2	0	2	-4	0	-1	1	
$Z = 3$		**Z_j**	**$\frac{9}{2}$**	**3**	**$\frac{3}{4}$**	**0**	
		$C_j - Z_j$	$-\frac{5}{2}$	0	$-\frac{3}{4}$	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 1$

Max $Z = 3$

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Find solution using Simplex(BigM) method
MAX Z = 2x1 + 5x2 + 7x3
subject to
3x1 + 2x2 + 4x3 <= 100
x1 + 4x2 + 2x3 <= 100
x1 + x2 + x3 <= 100
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = 2x_1 + 5x_2 + 7x_3$$

subject to

$$\begin{aligned} 3x_1 + 2x_2 + 4x_3 &\le 100 \\ x_1 + 4x_2 + 2x_3 &\le 100 \\ x_1 + x_2 + x_3 &\le 100 \end{aligned}$$

and $x_1, x_2, x_3 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\le$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = 2x_1 + 5x_2 + 7x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$\begin{aligned} 3x_1 + 2x_2 + 4x_3 + S_1 &= 100 \\ x_1 + 4x_2 + 2x_3 + S_2 &= 100 \\ x_1 + x_2 + x_3 + S_3 &= 100 \end{aligned}$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \ge 0$

Iteration-1		C_j	2	5	7	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{x_3}$
S_1	0	100	3	2	(4)	1	0	0	$\frac{100}{4} = 25 \rightarrow$
S_2	0	100	1	4	2	0	1	0	$\frac{100}{2} = 50$

S_3	0	100	1	1	1	0	0	1	$\frac{100}{1} = 100$
$\boldsymbol{Z = 0}$		Z_j	**0**	**0**	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	2	5	7 ↑	0	0	0	

Positive maximum $C_j - Z_j$ is 7 and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 25 and its row index is 1. So, the leaving basis variable is S_1.

∴ The pivot element is 4.

Entering $= x_3$, Departing $= S_1$, Key Element $= 4$

$R_1(\text{new}) = R_1(\text{old}) \div 4$

$R_2(\text{new}) = R_2(\text{old}) - 2R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - R_1(\text{new})$

Iteration-2		C_j	2	5	7	0	0	0	
$\boldsymbol{B}$	$\boldsymbol{C_B}$	$\boldsymbol{X_B}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	$\boldsymbol{S_3}$	**MinRatio** $\boldsymbol{\frac{X_B}{x_2}}$
x_3	7	25	$\frac{3}{4}$	$\frac{1}{2}$	1	$\frac{1}{4}$	0	0	$\frac{25}{\frac{1}{2}} = 50$
$\boldsymbol{S_2}$	0	50	$-\frac{1}{2}$	**(3)**	0	$-\frac{1}{2}$	1	0	$\frac{50}{3} = \frac{50}{3} \rightarrow$
S_3	0	75	$\frac{1}{4}$	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	1	$\frac{75}{\frac{1}{2}} = 150$
$\boldsymbol{Z = 175}$		Z_j	$\boldsymbol{\frac{21}{4}}$	$\boldsymbol{\frac{7}{2}}$	**7**	$\boldsymbol{\frac{7}{4}}$	**0**	**0**	
		$C_j - Z_j$	$-\frac{13}{4}$	$\frac{3}{2}$ ↑	0	$-\frac{7}{4}$	0	0	

Positive maximum $C_j - Z_j$ is $\frac{3}{2}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{50}{3}$ and its row index is 2. So, the leaving basis variable is S_2.

$\therefore$ The pivot element is 3.

Entering $= x_2$, Departing $= S_2$, Key Element $= 3$

R_2(new) $= R_2$(old) $\div 3$

R_1(new) $= R_1$(old)- $\frac{1}{2} R_2$(new)

R_3(new) $= R_3$(old)- $\frac{1}{2} R_2$(new)

Iteration-3		C_j	2	5	7	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**	**MinRatio**
x_3	7	$\frac{50}{3}$	$\frac{5}{6}$	0	1	$\frac{1}{3}$	$-\frac{1}{6}$	0	
x_2	5	$\frac{50}{3}$	$-\frac{1}{6}$	1	0	$-\frac{1}{6}$	$\frac{1}{3}$	0	
S_3	0	$\frac{200}{3}$	$\frac{1}{3}$	0	0	$-\frac{1}{6}$	$-\frac{1}{6}$	1	
$Z = 200$		Z_j	**5**	**5**	**7**	$\mathbf{\frac{3}{2}}$	$\mathbf{\frac{1}{2}}$	**0**	
		$C_j - Z_j$	-3	0	0	$-\frac{3}{2}$	$-\frac{1}{2}$	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = 0, x_2 = \frac{50}{3}, x_3 = \frac{50}{3}$

Max $Z = 200$

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Find solution using Simplex(BigM) method
MAX Z = 2x1 + x2
subject to
2x1 + x2 <= 10
2x1 <= 40
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 2x_1 + x_2$$

subject to

$$2x_1 + x_2 \le 10$$
$$2x_1 \le 40$$

and $x_1, x_2 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 2x_1 + x_2 + 0S_1 + 0S_2$$

subject to

$$2x_1 + x_2 + S_1 = 10$$
$$2x_1 + S_2 = 40$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	2	1	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	10	(2)	1	1	0	$\frac{10}{2} = 5 \rightarrow$
S_2	0	40	2	0	0	1	$\frac{40}{2} = 20$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	$2 \uparrow$	1	0	0	

Positive maximum C_j - Z_j is 2 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 5 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 2.

Entering $= x_1$, Departing $= S_1$, Key Element $= 2$

R_1(new) $= R_1$(old) $\div 2$

R_2(new) $= R_2$(old)- $2R_1$(new)

Iteration-2		C_j	2	1	0	0	
B	$\boldsymbol{C_B}$	$\boldsymbol{X_B}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	**MinRatio**
x_1	2	5	1	$\frac{1}{2}$	$\frac{1}{2}$	0	
S_2	0	30	0	-1	-1	1	
$\boldsymbol{Z = 10}$		$\boldsymbol{Z_j}$	**2**	**1**	**1**	**0**	
		C_j - Z_j	0	0	-1	0	

Since all C_j - $Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 5, x_2 = 0$

Max $Z = 10$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 2x2
subject to
2x1 + 2x2 <= 10
x1 + 3x2 <= 6
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to

$$2x_1 + 2x_2 \leq 10$$
$$x_1 + 3x_2 \leq 6$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 3x_1 + 2x_2 + 0S_1 + 0S_2$$

subject to

$$2x_1 + 2x_2 + S_1 = 10$$
$$x_1 + 3x_2 + S_2 = 6$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	3	2	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	10	(2)	2	1	0	$\frac{10}{2} = 5 \rightarrow$
S_2	0	6	1	3	0	1	$\frac{6}{1} = 6$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	$3 \uparrow$	2	0	0	

Positive maximum $C_j - Z_j$ is 3 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 5 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 2.

Entering $= x_1$, Departing $= S_1$, Key Element $= 2$

$R_1(\text{new}) = R_1(\text{old}) \div 2$

$R_2(\text{new}) = R_2(\text{old}) - R_1(\text{new})$

Iteration-2		C_j	3	2	0	0	
B	$\boldsymbol{C_B}$	$\boldsymbol{X_B}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	**MinRatio**
x_1	3	5	1	1	$\frac{1}{2}$	0	
S_2	0	1	0	2	$-\frac{1}{2}$	1	
$\boldsymbol{Z = 15}$		$\boldsymbol{Z_j}$	**3**	**3**	$\boldsymbol{\frac{3}{2}}$	**0**	
		$C_j - Z_j$	0	-1	$-\frac{3}{2}$	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 5, x_2 = 0$

Max $Z = 15$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 9x2
subject to
x1 + 4x2 <= 8
x1 + 2x2 <= 4
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 9x_2$$

subject to

$$x_1 + 4x_2 \le 8$$
$$x_1 + 2x_2 \le 4$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 3x_1 + 9x_2 + 0S_1 + 0S_2$$

subject to

$$x_1 + 4x_2 + S_1 = 8$$
$$x_1 + 2x_2 + S_2 = 4$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	3	9	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	8	1	4	1	0	$\frac{8}{4} = 2$
S_2	0	4	1	**(2)**	0	1	$\frac{4}{2} = 2 \rightarrow$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	3	$9 \uparrow$	0	0	

Positive maximum $C_j - Z_j$ is 9 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 2 and its row index is 2. So, the leaving basis variable is S_2.

$\therefore$ The pivot element is 2.

Entering $= x_2$, Departing $= S_2$, Key Element $= 2$

$R_2(\text{new}) = R_2(\text{old}) \div 2$

$R_1(\text{new}) = R_1(\text{old}) - 4R_2(\text{new})$

Iteration-2		C_j	3	9	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio**
S_1	0	0	-1	0	1	-2	
x_2	9	2	$\frac{1}{2}$	1	0	$\frac{1}{2}$	
$Z = 18$		Z_j	$\frac{9}{2}$	9	0	$\frac{9}{2}$	
		$C_j - Z_j$	$-\frac{3}{2}$	0	0	$-\frac{9}{2}$	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 2$

Max $Z = 18$

Solution is provided by AtoZmath.com

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + x2 + 2x3
subject to
x1 + x2 + x3 <= 2
2x1 + x2 + x3 <= 3
x1 + 2x2 + 2x3 <= 4
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + x_2 + 2x_3$$

subject to

$$x_1 + x_2 + x_3 \le 2$$
$$2x_1 + x_2 + x_3 \le 3$$
$$x_1 + 2x_2 + 2x_3 \le 4$$

and $x_1, x_2, x_3 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\le$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = 3x_1 + x_2 + 2x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$x_1 + x_2 + x_3 + S_1 = 2$$
$$2x_1 + x_2 + x_3 + S_2 = 3$$
$$x_1 + 2x_2 + 2x_3 + S_3 = 4$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \ge 0$

Iteration-1		C_j	3	1	2	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_1}$
S_1	0	2	1	1	1	1	0	0	$\frac{2}{1} = 2$
S_2	0	3	(2)	1	1	0	1	0	$\frac{3}{2} = \frac{3}{2} \rightarrow$

S_3	0	4	1	2	2	0	0	1	$\frac{4}{1} = 4$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	3 ↑	1	2	0	0	0	

Positive maximum $C_j - Z_j$ is 3 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is $\frac{3}{2}$ and its row index is 2. So, the leaving basis variable is S_2.

∴ The pivot element is 2.

Entering $= x_1$, Departing $= S_2$, Key Element $= 2$

$R_2(\text{new}) = R_2(\text{old}) \div 2$

$R_1(\text{new}) = R_1(\text{old}) - R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - R_2(\text{new})$

Iteration-2		C_j	3	1	2	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{x_3}$
S_1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\left(\frac{1}{2}\right)$	1	$-\frac{1}{2}$	0	$\frac{\frac{1}{2}}{\frac{1}{2}} = 1$ →
x_1	3	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{\frac{3}{2}}{\frac{1}{2}} = 3$
S_3	0	$\frac{5}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}$	0	$-\frac{1}{2}$	1	$\frac{\frac{5}{2}}{\frac{3}{2}} = \frac{5}{3}$
$Z = \frac{9}{2}$		Z_j	**3**	$\frac{3}{2}$	$\frac{3}{2}$	**0**	$\frac{3}{2}$	**0**	
		$C_j - Z_j$	0	$-\frac{1}{2}$	$\frac{1}{2}$ ↑	0	$-\frac{3}{2}$	0	

Positive maximum $C_j - Z_j$ is $\frac{1}{2}$ and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 1 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is $\frac{1}{2}$.

Entering $= x_3$, Departing $= S_1$, Key Element $= \frac{1}{2}$

$R_1(\text{new}) = R_1(\text{old}) \times 2$

$R_2(\text{new}) = R_2(\text{old}) - \frac{1}{2} R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{3}{2} R_1(\text{new})$

Iteration-3		C_j	3	1	2	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**	**MinRatio**
x_3	2	1	0	1	1	2	-1	0	
x_1	3	1	1	0	0	-1	1	0	
S_3	0	1	0	0	0	-3	1	1	
$Z = 5$		Z_j	**3**	**2**	**2**	**1**	**1**	**0**	
		$C_j - Z_j$	0	-1	0	-1	-1	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 1, x_2 = 0, x_3 = 1$

Max $Z = 5$

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Find solution using Simplex(BigM) method
MAX Z = 4x1 + 3x2
subject to
2x1 + x2 <= 1000
x1 + x2 <= 800
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 4x_1 + 3x_2$$

subject to

$$2x_1 + x_2 \le 1000$$
$$x_1 + x_2 \le 800$$

and $x_1, x_2 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 4x_1 + 3x_2 + 0S_1 + 0S_2$$

subject to

$$2x_1 + x_2 + S_1 = 1000$$
$$x_1 + x_2 + S_2 = 800$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	4	3	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	1000	**(2)**	1	1	0	$\frac{1000}{2} = 500 \rightarrow$
S_2	0	800	1	1	0	1	$\frac{800}{1} = 800$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	4 $\uparrow$	3	0	0	

Positive maximum C_j - Z_j is 4 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 500 and its row index is 1. So, the leaving basis variable is S_1.

∴ The pivot element is 2.

Entering $= x_1$, Departing $= S_1$, Key Element $= 2$

$R_1(\text{new}) = R_1(\text{old}) \div 2$

$R_2(\text{new}) = R_2(\text{old}) - R_1(\text{new})$

Iteration-2		C_j	4	3	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio** $\frac{X_B}{x_2}$
x_1	4	500	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{500}{\frac{1}{2}} = 1000$
S_2	0	300	0	$\left(\frac{1}{2}\right)$	$-\frac{1}{2}$	1	$\frac{300}{\frac{1}{2}} = 600 \rightarrow$
$Z = 2000$		Z_j	**4**	**2**	**2**	**0**	
		$C_j - Z_j$	0	1 ↑	-2	0	

Positive maximum C_j - Z_j is 1 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 600 and its row index is 2. So, the leaving basis variable is S_2.

∴ The pivot element is $\frac{1}{2}$.

Entering $= x_2$, Departing $= S_2$, Key Element $= \frac{1}{2}$

$R_2(\text{new}) = R_2(\text{old}) \times 2$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{2} R_2(\text{new})$

Iteration-3		C_j	4	3	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio**
x_1	4	200	1	0	1	-1	

x_2	3	600	0	1	-1	2	
Z = 2600		Z_j	**4**	**3**	**1**	**2**	
		$C_j - Z_j$	0	0	-1	-2	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 200, x_2 = 600$

Max $Z = 2600$

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Find solution using Simplex(BigM) method
MAX Z = 4x1 + 8x2
subject to
2x1 + 4x2 <= 4
3x1 + 6x2 <= 6
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 4x_1 + 8x_2$$

subject to

$$2x_1 + 4x_2 \leq 4$$

$$3x_1 + 6x_2 \leq 6$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 4x_1 + 8x_2 + 0S_1 + 0S_2$$

subject to

$$2x_1 + 4x_2 + S_1 = 4$$

$$3x_1 + 6x_2 + S_2 = 6$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	4	8	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	4	2	4	1	0	$\frac{4}{4} = 1$
S_2	0	6	3	(6)	0	1	$\frac{6}{6} = 1 \rightarrow$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	4	$8 \uparrow$	0	0	

Positive maximum $C_j - Z_j$ is 8 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 1 and its row index is 2. So, the leaving basis variable is S_2.

$\therefore$ The pivot element is 6.

Entering $= x_2$, Departing $= S_2$, Key Element $= 6$

$R_2(\text{new}) = R_2(\text{old}) \div 6$

$R_1(\text{new}) = R_1(\text{old}) - 4R_2(\text{new})$

Iteration-2		C_j	4	8	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio**
S_1	0	0	0	0	1	$-\frac{2}{3}$	
x_2	8	1	$\frac{1}{2}$	1	0	$\frac{1}{6}$	
$Z = 8$		Z_j	**4**	**8**	**0**	$\frac{4}{3}$	
		$C_j - Z_j$	0	0	0	$-\frac{4}{3}$	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 1$

Max $Z = 8$

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Find solution using Simplex(BigM) method
MAX Z = 5x1 + 16x2
subject to
2x1 + 5x2 <= 10000
3x1 + 4x2 <= 15000
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 5x_1 + 16x_2$$

subject to

$$2x_1 + 5x_2 \le 10000$$
$$3x_1 + 4x_2 \le 15000$$

and $x_1, x_2 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 5x_1 + 16x_2 + 0S_1 + 0S_2$$

subject to

$$2x_1 + 5x_2 + S_1 = 10000$$
$$3x_1 + 4x_2 + S_2 = 15000$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	5	16	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	MinRatio $\frac{X_B}{x_2}$
S_1	0	10000	2	(5)	1	0	$\frac{10000}{5} = 2000 \rightarrow$
S_2	0	15000	3	4	0	1	$\frac{15000}{4} = 3750$
$Z = 0$		Z_j	0	0	0	0	
		$C_j - Z_j$	5	16 $\uparrow$	0	0	

Positive maximum $C_j - Z_j$ is 16 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 2000 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 5.

Entering $= x_2$, Departing $= S_1$, Key Element $= 5$

$R_1(\text{new}) = R_1(\text{old}) \div 5$

$R_2(\text{new}) = R_2(\text{old}) - 4R_1(\text{new})$

Iteration-2		C_j	5	16	0	0	
B	$\boldsymbol{C_B}$	$\boldsymbol{X_B}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	**MinRatio**
x_2	16	2000	$\frac{2}{5}$	1	$\frac{1}{5}$	0	
S_2	0	7000	$\frac{7}{5}$	0	$-\frac{4}{5}$	1	
$\boldsymbol{Z = 32000}$		$\boldsymbol{Z_j}$	$\boldsymbol{\frac{32}{5}}$	**16**	$\boldsymbol{\frac{16}{5}}$	**0**	
		$C_j - Z_j$	$-\frac{7}{5}$	0	$-\frac{16}{5}$	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 2000$

Max $Z = 32000$

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Find solution using Simplex(BigM) method
MAX Z = 6x1 + 8x2
subject to
30x1 + 20x2 <= 300
5x1 + 10x2 <= 110
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 6x_1 + 8x_2$$

subject to

$$30x_1 + 20x_2 \le 300$$
$$5x_1 + 10x_2 \le 110$$

and $x_1, x_2 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 6x_1 + 8x_2 + 0S_1 + 0S_2$$

subject to

$$30x_1 + 20x_2 + S_1 = 300$$
$$5x_1 + 10x_2 + S_2 = 110$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	6	8	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	MinRatio $\frac{X_B}{x_2}$
S_1	0	300	30	20	1	0	$\frac{300}{20} = 15$
S_2	0	110	5	**(10)**	0	1	$\frac{110}{10} = 11 \rightarrow$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	6	$8 \uparrow$	0	0	

Positive maximum C_j - Z_j is 8 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 11 and its row index is 2. So, the leaving basis variable is S_2.

∴ The pivot element is 10.

Entering $= x_2$, Departing $= S_2$, Key Element $= 10$

R_2(new) = R_2(old) ÷ 10

R_1(new) = R_1(old)-20R_2(new)

Iteration-2		C_j	6	8	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	80	**(20)**	0	1	-2	$\frac{80}{20} = 4 \rightarrow$
x_2	8	11	$\frac{1}{2}$	1	0	$\frac{1}{10}$	$\frac{11}{\frac{1}{2}} = 22$
$Z = 88$		**Z_j**	**4**	**8**	**0**	$\frac{4}{5}$	
		C_j - Z_j	2 ↑	0	0	$-\frac{4}{5}$	

Positive maximum C_j - Z_j is 2 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 4 and its row index is 1. So, the leaving basis variable is S_1.

∴ The pivot element is 20.

Entering $= x_1$, Departing $= S_1$, Key Element $= 20$

R_1(new) = R_1(old) ÷ 20

R_2(new) = R_2(old)-$\frac{1}{2}R_1$(new)

Iteration-3		C_j	6	8	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio**
x_1	6	4	1	0	$\frac{1}{20}$	$-\frac{1}{10}$	

x_2	8	9	0	1	$-\frac{1}{40}$	$\frac{3}{20}$	
$Z = 96$		Z_j	**6**	**8**	$\frac{1}{10}$	$\frac{3}{5}$	
		$C_j - Z_j$	0	0	$-\frac{1}{10}$	$-\frac{3}{5}$	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 4, x_2 = 9$

Max $Z = 96$

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Find solution using Simplex(BigM) method
MAX Z = 7x1 + 6x2
subject to
6x1 + 7x2 <= 12
5x2 <= 30
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 7x_1 + 6x_2$$

subject to

$$6x_1 + 7x_2 \leq 12$$
$$5x_2 \leq 30$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 7x_1 + 6x_2 + 0S_1 + 0S_2$$

subject to

$$6x_1 + 7x_2 + S_1 = 12$$
$$5x_2 + S_2 = 30$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	7	6	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	12	(6)	7	1	0	$\frac{12}{6} = 2 \rightarrow$
S_2	0	30	0	5	0	1	---
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	$7 \uparrow$	6	0	0	

Positive maximum $C_j - Z_j$ is 7 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 2 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 6.

Entering $= x_1$, Departing $= S_1$, Key Element $= 6$

R_1(new) $= R_1$(old) $\div 6$

R_2(new) $= R_2$(old)

Iteration-2		C_j	7	6	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio**
x_1	7	2	1	$\frac{7}{6}$	$\frac{1}{6}$	0	
S_2	0	30	0	5	0	1	
$Z = 14$		**Z_j**	**7**	**$\frac{49}{6}$**	**$\frac{7}{6}$**	**0**	
		$C_j - Z_j$	0	$-\frac{13}{6}$	$-\frac{7}{6}$	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 2, x_2 = 0$

Max $Z = 14$

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Find solution using Simplex(BigM) method
MAX Z = 120x1 + 110x2
subject to
5x1 + 2x2 <= 100
3x1 + 3x2 <= 50
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 120x_1 + 110x_2$$

subject to

$$5x_1 + 2x_2 \leq 100$$

$$3x_1 + 3x_2 \leq 50$$

and $x_1, x_2 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 120x_1 + 110x_2 + 0S_1 + 0S_2$$

subject to

$$5x_1 + 2x_2 + S_1 = 100$$

$$3x_1 + 3x_2 + S_2 = 50$$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration-1		C_j	120	110	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	100	5	2	1	0	$\frac{100}{5} = 20$
S_2	0	50	(3)	3	0	1	$\frac{50}{3} = \frac{50}{3} \rightarrow$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	120 $\uparrow$	110	0	0	

Positive maximum $C_j - Z_j$ is 120 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is $\frac{50}{3}$ and its row index is 2. So, the leaving basis variable is S_2.

$\therefore$ The pivot element is 3.

Entering $= x_1$, Departing $= S_2$, Key Element $= 3$

$R_2(\text{new}) = R_2(\text{old}) \div 3$

$R_1(\text{new}) = R_1(\text{old}) - 5R_2(\text{new})$

Iteration-2		C_j	120	110	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**MinRatio**
S_1	0	$\frac{50}{3}$	0	-3	1	$-\frac{5}{3}$	
x_1	120	$\frac{50}{3}$	1	1	0	$\frac{1}{3}$	
$Z = 2000$		**Z_j**	**120**	**120**	**0**	**40**	
		$C_j - Z_j$	0	-10	0	-40	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = \frac{50}{3}, x_2 = 0$

Max $Z = 2000$

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Find solution using Simplex(BigM) method
MAX Z = 500x1 + 600x2 + 1200x3
subject to
2x1 + 4x2 + 6x3 <= 160
3x1 + 2x2 + 4x3 <= 120
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = 500x_1 + 600x_2 + 1200x_3$$

subject to

$$2x_1 + 4x_2 + 6x_3 \leq 160$$

$$3x_1 + 2x_2 + 4x_3 \leq 120$$

and $x_1, x_2, x_3 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 500x_1 + 600x_2 + 1200x_3 + 0S_1 + 0S_2$$

subject to

$$2x_1 + 4x_2 + 6x_3 + S_1 = 160$$

$$3x_1 + 2x_2 + 4x_3 + S_2 = 120$$

and $x_1, x_2, x_3, S_1, S_2 \geq 0$

Iteration-1		C_j	500	600	1200	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	**MinRatio** $\frac{X_B}{x_3}$
S_1	0	160	2	4	(6)	1	0	$\frac{160}{6} = \frac{80}{3} \rightarrow$
S_2	0	120	3	2	4	0	1	$\frac{120}{4} = 30$
$Z = 0$		Z_j	0	0	0	0	0	
		$C_j - Z_j$	500	600	1200 $\uparrow$	0	0	

Positive maximum $C_j - Z_j$ is 1200 and its column index is 3. So, the entering variable is x_3.

Minimum ratio is $\frac{80}{3}$ and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 6.

Entering $= x_3$, Departing $= S_1$, Key Element $= 6$

$R_1(\text{new}) = R_1(\text{old}) \div 6$

$R_2(\text{new}) = R_2(\text{old}) - 4R_1(\text{new})$

Iteration-2		C_j	500	600	1200	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**MinRatio** $\frac{X_B}{x_1}$
x_3	1200	$\frac{80}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{1}{6}$	0	$\frac{\frac{80}{3}}{\frac{1}{3}} = 80$
S_2	0	$\frac{40}{3}$	$\left(\frac{5}{3}\right)$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	1	$\frac{\frac{40}{3}}{\frac{5}{3}} = 8 \rightarrow$
$Z = 32000$		Z_j	**400**	**800**	**1200**	**200**	**0**	
		$C_j - Z_j$	100 $\uparrow$	-200	0	-200	0	

Positive maximum $C_j - Z_j$ is 100 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 8 and its row index is 2. So, the leaving basis variable is S_2.

$\therefore$ The pivot element is $\frac{5}{3}$.

Entering $= x_1$, Departing $= S_2$, Key Element $= \frac{5}{3}$

$R_2(\text{new}) = R_2(\text{old}) \times \frac{3}{5}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{1}{3}R_2(\text{new})$

Iteration-3		C_j	500	600	1200	0	0	

B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	MinRatio
x_3	1200	24	0	$\frac{4}{5}$	1	$\frac{3}{10}$	$-\frac{1}{5}$	
x_1	500	8	1	$-\frac{2}{5}$	0	$-\frac{2}{5}$	$\frac{3}{5}$	
$Z = 32800$		Z_j	**500**	**760**	**1200**	**160**	**60**	
		$C_j - Z_j$	0	-160	0	-160	-60	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 8, x_2 = 0, x_3 = 24$

Max $Z = 32800$

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Find solution using Simplex(BigM) method
MAX Z = x1 - x2 + 2x3
subject to
x1 + x2 + x3 <= 4
x1 - 2x2 + x3 <= 6
3x1 + 2x2 + x3 <= 0
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = x_1 - x_2 + 2x_3$$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 4 \\ x_1 - 2x_2 + x_3 &\leq 6 \\ 3x_1 + 2x_2 + x_3 &\leq 0 \end{aligned}$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\leq$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\leq$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\leq$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = x_1 - x_2 + 2x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 + S_1 &= 4 \\ x_1 - 2x_2 + x_3 + S_2 &= 6 \\ 3x_1 + 2x_2 + x_3 + S_3 &= 0 \end{aligned}$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iteration-1		C_j	1	-1	2	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_3}$
S_1	0	4	1	1	1	1	0	0	$\frac{4}{1} = 4$
S_2	0	6	1	-2	1	0	1	0	$\frac{6}{1} = 6$

S_3	0	0	3	2	(1)	0	0	1	$\frac{0}{1} = 0 \rightarrow$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	1	-1	2 $\uparrow$	0	0	0	

Positive maximum $C_j - Z_j$ is 2 and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 0 and its row index is 3. So, the leaving basis variable is S_3.

$\therefore$ The pivot element is 1.

Entering $= x_3$, Departing $= S_3$, Key Element $= 1$

$R_3(\text{new}) = R_3(\text{old})$

$R_1(\text{new}) = R_1(\text{old}) - R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - R_3(\text{new})$

Iteration-2		C_j	1	-1	2	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**S_1**	**S_2**	**S_3**	**MinRatio**
S_1	0	4	-2	-1	0	1	0	-1	
S_2	0	6	-2	-4	0	0	1	-1	
x_3	2	0	3	2	1	0	0	1	
$Z = 0$		Z_j	**6**	**4**	**2**	**0**	**0**	**2**	
		$C_j - Z_j$	-5	-5	0	0	0	-2	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 0, x_3 = 0$

Max $Z = 0$

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Find solution using Simplex(BigM) method
MAX Z = x1 + x2 + 2x3
subject to
3x1 + 2x2 + x3 <= 2
2x1 + 2x2 + x3 <= 3
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = x_1 + x_2 + 2x_3$$

subject to

$$3x_1 + 2x_2 + x_3 \le 2$$
$$2x_1 + 2x_2 + x_3 \le 3$$

and $x_1, x_2, x_3 \ge 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = x_1 + x_2 + 2x_3 + 0S_1 + 0S_2$$

subject to

$$3x_1 + 2x_2 + x_3 + S_1 = 2$$
$$2x_1 + 2x_2 + x_3 + S_2 = 3$$

and $x_1, x_2, x_3, S_1, S_2 \ge 0$

Iteration-1		C_j	1	1	2	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	**MinRatio** $\frac{X_B}{x_3}$
S_1	0	2	3	2	**(1)**	1	0	$\frac{2}{1} = 2 \rightarrow$
S_2	0	3	2	2	1	0	1	$\frac{3}{1} = 3$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	1	1	$2 \uparrow$	0	0	

Positive maximum C_j - Z_j is 2 and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 2 and its row index is 1. So, the leaving basis variable is S_1.

∴ The pivot element is 1.

Entering $= x_3$, Departing $= S_1$, Key Element $= 1$

R_1(new) = R_1(old)

R_2(new) = R_2(old)-R_1(new)

Iteration-2		C_j	1	1	2	0	0	
B	$\boldsymbol{C_B}$	$\boldsymbol{X_B}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	**MinRatio**
x_3	2	2	3	2	1	1	0	
S_2	0	1	-1	0	0	-1	1	
$\boldsymbol{Z = 4}$		Z_j	**6**	**4**	**2**	**2**	**0**	
		C_j - Z_j	-5	-3	0	-2	0	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 0, x_3 = 2$

Max $Z = 4$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 2x2 + x3
subject to
x1 + 2x2 + x3 <= 12
x1 + 3x2 <= 18
2x1 + 4x3 <= 22
and x1,x2,x3 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 2x_2 + x_3$$

subject to

$$x_1 + 2x_2 + x_3 \le 12$$
$$x_1 + 3x_2 \le 18$$
$$2x_1 + 4x_3 \le 22$$

and $x_1, x_2, x_3 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\le$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = 3x_1 + 2x_2 + x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$x_1 + 2x_2 + x_3 + S_1 = 12$$
$$x_1 + 3x_2 + S_2 = 18$$
$$2x_1 + 4x_3 + S_3 = 22$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \ge 0$

Iteration-1		C_j	3	2	1	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_1}$
S_1	0	12	1	2	1	1	0	0	$\frac{12}{1} = 12$
S_2	0	18	1	3	0	0	1	0	$\frac{18}{1} = 18$

S_3	0	22	**(2)**	0	4	0	0	1	$\frac{22}{2} = 11 \rightarrow$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	$3 \uparrow$	2	1	0	0	0	

Positive maximum $C_j - Z_j$ is 3 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 11 and its row index is 3. So, the leaving basis variable is S_3.

$\therefore$ The pivot element is 2.

Entering $= x_1$, Departing $= S_3$, Key Element $= 2$

$R_3(\text{new}) = R_3(\text{old}) \div 2$

$R_1(\text{new}) = R_1(\text{old}) - R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - R_3(\text{new})$

Iteration-2		C_j	3	2	1	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	1	0	**(2)**	-1	1	0	$-\frac{1}{2}$	$\frac{1}{2} = \frac{1}{2} \rightarrow$
S_2	0	7	0	3	-2	0	1	$-\frac{1}{2}$	$\frac{7}{3} = \frac{7}{3}$
x_1	3	11	1	0	2	0	0	$\frac{1}{2}$	---
$Z = 33$		Z_j	**3**	**0**	**6**	**0**	**0**	$\frac{3}{2}$	
		$C_j - Z_j$	0	$2 \uparrow$	-5	0	0	$-\frac{3}{2}$	

Positive maximum $C_j - Z_j$ is 2 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is $\frac{1}{2}$ and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 2.

Entering $= x_2$, Departing $= S_1$, Key Element $= 2$

R_1(new) $= R_1$(old) $\div 2$

R_2(new) $= R_2$(old)$- 3R_1$(new)

R_3(new) $= R_3$(old)

Iteration-3		C_j	3	2	1	0	0	0	
B	$\boldsymbol{C_B}$	$\boldsymbol{X_B}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	$\boldsymbol{S_3}$	**MinRatio**
x_2	2	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{4}$	
S_2	0	$\frac{11}{2}$	0	0	$-\frac{1}{2}$	$-\frac{3}{2}$	1	$\frac{1}{4}$	
x_1	3	11	1	0	2	0	0	$\frac{1}{2}$	
$\boldsymbol{Z = 34}$		Z_j	**3**	**2**	**5**	**1**	**0**	**1**	
		$C_j - Z_j$	0	0	-4	-1	0	-1	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 11, x_2 = \frac{1}{2}, x_3 = 0$

Max $Z = 34$

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Find solution using Simplex(BigM) method
MAX Z = 3x1 + 2x2
subject to
4x1 + 3x2 <= 12
4x1 + x2 <= 8
4x1 - x2 <= 8
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to

$$4x_1 + 3x_2 \le 12$$
$$4x_1 + x_2 \le 8$$
$$4x_1 - x_2 \le 8$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

3. As the constraint 3 is of type ' $\le$ ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$4x_1 + 3x_2 + S_1 = 12$$
$$4x_1 + x_2 + S_2 = 8$$
$$4x_1 - x_2 + S_3 = 8$$

and $x_1, x_2, S_1, S_2, S_3 \ge 0$

Iteration-1		C_j	3	2	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_1}$
S_1	0	12	4	3	1	0	0	$\frac{12}{4} = 3$
S_2	0	8	4	1	0	1	0	$\frac{8}{4} = 2$

S_3	0	8	(4)	-1	0	0	1	$\frac{8}{4}=2$ →
$Z=0$		Z_j	0	0	0	0	0	
		$C_j - Z_j$	3 ↑	2	0	0	0	

Positive maximum $C_j - Z_j$ is 3 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 2 and its row index is 3. So, the leaving basis variable is S_3.

∴ The pivot element is 4.

Entering $= x_1$, Departing $= S_3$, Key Element $= 4$

$R_3(\text{new}) = R_3(\text{old}) \div 4$

$R_1(\text{new}) = R_1(\text{old}) - 4R_3(\text{new})$

$R_2(\text{new}) = R_2(\text{old}) - 4R_3(\text{new})$

Iteration-2		C_j	3	2	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_2}$
S_1	0	4	0	4	1	0	-1	$\frac{4}{4}=1$
S_2	0	0	0	(2)	0	1	-1	$\frac{0}{2}=0$ →
x_1	3	2	1	$-\frac{1}{4}$	0	0	$\frac{1}{4}$	---
$Z=6$		Z_j	3	$-\frac{3}{4}$	0	0	$\frac{3}{4}$	
		$C_j - Z_j$	0	$\frac{11}{4}$ ↑	0	0	$-\frac{3}{4}$	

Positive maximum $C_j - Z_j$ is $\frac{11}{4}$ and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is S_2.

∴ The pivot element is 2.

Entering $= x_2$, Departing $= S_2$, Key Element $= 2$

$R_2(\text{new}) = R_2(\text{old}) \div 2$

$R_1(\text{new}) = R_1(\text{old}) - 4R_2(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) + \frac{1}{4}R_2(\text{new})$

Iteration-3		C_j	3	2	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{S_3}$
S_1	0	4	0	0	1	-2	**(1)**	$\frac{4}{1} = 4 \rightarrow$
x_2	2	0	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	---
x_1	3	2	1	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{2}{\frac{1}{8}} = 16$
$Z = 6$		Z_j	**3**	**2**	**0**	$\frac{11}{8}$	$-\frac{5}{8}$	
		$C_j - Z_j$	0	0	0	$-\frac{11}{8}$	$\frac{5}{8} \uparrow$	

Positive maximum $C_j - Z_j$ is $\frac{5}{8}$ and its column index is 5. So, the entering variable is S_3.

Minimum ratio is 4 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 1.

Entering $= S_3$, Departing $= S_1$, Key Element $= 1$

$R_1(\text{new}) = R_1(\text{old})$

$R_2(\text{new}) = R_2(\text{old}) + \frac{1}{2}R_1(\text{new})$

$R_3(\text{new}) = R_3(\text{old}) - \frac{1}{8}R_1(\text{new})$

Iteration-4		C_j	3	2	0	0	0	
B								**MinRatio**

	C_B	X_B	x_1	x_2	S_1	S_2	S_3	
S_3	0	4	0	0	1	-2	1	
x_2	2	2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	
x_1	3	$\frac{3}{2}$	1	0	$-\frac{1}{8}$	$\frac{3}{8}$	0	
$Z = \frac{17}{2}$		Z_j	**3**	**2**	$\frac{5}{8}$	$\frac{1}{8}$	**0**	
		$C_j - Z_j$	0	0	$-\frac{5}{8}$	$-\frac{1}{8}$	0	

Since all $C_j - Z_j \le 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = \frac{3}{2}, x_2 = 2$

Max $Z = \frac{17}{2}$

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Find solution using Simplex(BigM) method
MAX Z = 5x1 + 6x2
subject to
2x1 + 5x2 <= 10000
3x1 + 4x2 <= 15000
and x1,x2 >= 0

Solution:
Problem is

$$\text{Max } Z = 5x_1 + 6x_2$$

subject to

$$2x_1 + 5x_2 \le 10000$$
$$3x_1 + 4x_2 \le 15000$$

and $x_1, x_2 \ge 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint 1 is of type ' $\le$ ' we should add slack variable S_1

2. As the constraint 2 is of type ' $\le$ ' we should add slack variable S_2

After introducing slack variables

$$\text{Max } Z = 5x_1 + 6x_2 + 0S_1 + 0S_2$$

subject to

$$2x_1 + 5x_2 + S_1 = 10000$$
$$3x_1 + 4x_2 + S_2 = 15000$$

and $x_1, x_2, S_1, S_2 \ge 0$

Iteration-1		C_j	5	6	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	10000	2	(5)	1	0	$\frac{10000}{5} = 2000 \rightarrow$
S_2	0	15000	3	4	0	1	$\frac{15000}{4} = 3750$
$Z = 0$		Z_j	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	5	6 $\uparrow$	0	0	

Positive maximum $C_j - Z_j$ is 6 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 2000 and its row index is 1. So, the leaving basis variable is S_1.

$\therefore$ The pivot element is 5.

Entering $= x_2$, Departing $= S_1$, Key Element $= 5$

$R_1(\text{new}) = R_1(\text{old}) \div 5$

$R_2(\text{new}) = R_2(\text{old}) - 4R_1(\text{new})$

Iteration-2		C_j	5	6	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	**MinRatio** $\frac{X_B}{x_1}$
x_2	6	2000	$\frac{2}{5}$	1	$\frac{1}{5}$	0	$\frac{2000}{\frac{2}{5}} = 5000$
S_2	0	7000	$\left(\frac{7}{5}\right)$	0	$-\frac{4}{5}$	1	$\frac{7000}{\frac{7}{5}} = 5000 \rightarrow$
$Z = 12000$		Z_j	$\frac{12}{5}$	6	$\frac{6}{5}$	0	
		$C_j - Z_j$	$\frac{13}{5} \uparrow$	0	$-\frac{6}{5}$	0	

Positive maximum $C_j - Z_j$ is $\frac{13}{5}$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 5000 and its row index is 2. So, the leaving basis variable is S_2.

$\therefore$ The pivot element is $\frac{7}{5}$.

Entering $= x_1$, Departing $= S_2$, Key Element $= \frac{7}{5}$

$R_2(\text{new}) = R_2(\text{old}) \times \frac{5}{7}$

$R_1(\text{new}) = R_1(\text{old}) - \frac{2}{5}R_2(\text{new})$

Iteration-3		C_j	5	6	0	0	

B	C_B	X_B	x_1	x_2	S_1	S_2	MinRatio $\frac{X_B}{S_1}$
x_2	6	0	0	1	$\left(\frac{3}{7}\right)$	$-\frac{2}{7}$	$\frac{0}{\frac{3}{7}} = 0 \rightarrow$
x_1	5	5000	1	0	$-\frac{4}{7}$	$\frac{5}{7}$	---
$Z = 25000$		Z_j	5	6	$-\frac{2}{7}$	$\frac{13}{7}$	
		$C_j - Z_j$	0	0	$\frac{2}{7} \uparrow$	$-\frac{13}{7}$	

Positive maximum $C_j - Z_j$ is $\frac{2}{7}$ and its column index is 3. So, the entering variable is S_1.

Minimum ratio is 0 and its row index is 1. So, the leaving basis variable is x_2.

$\therefore$ The pivot element is $\frac{3}{7}$.

Entering $= S_1$, Departing $= x_2$, Key Element $= \frac{3}{7}$

$R_1(\text{new}) = R_1(\text{old}) \times \frac{7}{3}$

$R_2(\text{new}) = R_2(\text{old}) + \frac{4}{7} R_1(\text{new})$

Iteration-4		C_j	5	6	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	MinRatio
S_1	0	0	0	$\frac{7}{3}$	1	$-\frac{2}{3}$	
x_1	5	5000	1	$\frac{4}{3}$	0	$\frac{1}{3}$	
$Z = 25000$		Z_j	5	$\frac{20}{3}$	0	$\frac{5}{3}$	
		$C_j - Z_j$	0	$-\frac{2}{3}$	0	$-\frac{5}{3}$	

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 5000, x_2 = 0$

Max $Z = 25000$

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