

1. A manufacturing company is engaged in producing three types of product A, B, C. The production department produces each day, components sufficient to make 50 units of A, 25 units of B and 30 units of C. The management is confronted with the problem of optimizing the daily production of the products in the assembly department, where only 100 man hours are available daily for assembling the products. The following additional information is available.

Type product	Profit distribution per unit of product (R.S.)	Assembly time per period (hours)
A	12	0.8
B	20	1.7
C	45	2.5

The company has a daily order commitment for 20 units of products A and a total of 15 units of products B and C. Formulate this problem as an L.P. model so as to maximize the total profit.

sol ⁿ :	Products	A	B	C
	Profit (R.S.)	12	20	45

Sufficient components	50	25	30
Assembling time	0.8	1.7	2.5 $\rightarrow 100$ min h
Minimum	20	15	

Components

Let,
 x_1 = no. of units of A product
 x_2 = no. of units of B product
 x_3 = no. of units of C product

Maximize $P = 12x_1 + 20x_2 + 45x_3$

$$x_1 \leq 50$$

$$x_2 \leq 25$$

$$x_3 \leq 30$$

$$x_1 \geq 20$$

$$x_2 + x_3 \geq 15$$

$$0.8x_1 + 1.7x_2 + 2.5x_3 \leq 100$$

Where $x_1, x_2, x_3 \geq 0$

2 An electronic company is engaged in the production to two compartment C_1 and C_2 that are used in radio sets. Each unit of C_1 costs the company Rs. 5 wages and Rs. 5 in material, while each unit of C_2 costs the company Rs. 25 in wages and Rs. 15 in material. The company sells both products on one period credit terms, but the company's labour and material expenses must be paid in cash. The selling price of C_1 is Rs. 30 per unit and of C_2 it is Rs. 70 per unit. Because of the company's strong monopoly in these components, it is prevailing prices, as many units of as it produces. The company's production capacity is however limited by two considerations. First, at the beginning of the period 1, the company has an initial balance of Rs. 4000 (cash plus bank credit plus collection from past credit sales). Second, the company has available in each period 2000 h. of machine time and 1,400 hours of assembly time. ^{each C_1} requires 2 hours of machine time and 3 hours of assembly time. Where as the production of each C_2 requires 2 hours of machine time and 3 hours of assembly time. Formulate this problem as an L.P. model so as to maximize the total profit to the company.

Sol. :

	C_1	C_2
costs : material + wages	5+5	15+25 $\rightarrow$ 4000 Rs.

Selling price	30/unit	70/unit
---------------	---------	---------

machine time	3 hour	2 hour $\rightarrow$ total 2000 h available
--------------	--------	---

assembly time	2 hour	3 hour $\rightarrow$ total 1400 h. available
---------------	--------	--

let.

 x_1 = no. of units of C_1 produced x_2 = no. of units of C_2 produced

Profit = selling price - cost price

$$\text{Maximize } P = (30x_1 + 70x_2) - (10x_1 + 40x_2)$$

$$= 20x_1 + 30x_2$$

$$\text{Money Balance} = 10x_1 + 40x_2 \leq 4000$$

$$\text{Machine Time} = 3x_1 + 2x_2 \leq 2000$$

$$\text{Assembly Time} = 2x_1 + 3x_2 \leq 1400$$

Where $x_1, x_2 \geq 0$.

3. A company has two grades of inspectors 1. and 2. the members of which are to be assigned for a quality control inspection. It is required that at least 2000 pieces be inspected per 8 hours day. Grade 1 inspectors can check pieces at the rate of 40 per hour with an accuracy of 97 percent. Grade 2 inspectors can check pieces at the rate of 30 per hour with an accuracy of 95 percent.

The wage rate of a grade 1 inspector is RS. 500 per hour while rate of a grade 2 inspector is 400 per hour. An error made by an inspector costs RS. 200 to the company. There are only nine grade 1 inspectors and eleven grade 2 inspectors available to the company. The company wishes to assign work to total cost of the inspection. formulate this problem as an L.P. model so as to minimize daily inspection cost.

sol ⁿ :		Grade 1	Grade 2
No. of pieces (per day)		8×40	$8 \times 30 \rightarrow 200$
Wage rate (per day)		$8(500)$	$(8)(400)$
Average no. of errors		$(0.03)(320)$	$(0.05)(240)$
per day error cost		1920 RS.	2400 RS.

Let,

x_1 = no. of grade 1 inspectors assign per day.

x_2 = no. of grade 2 inspectors assign per day.

* Objective function:

cost of inspection = cost of error + Inspector salary

cost of grade 1 = $1920 + 4000 = 5920$

cost of grade 2 = $2400 + 3200 = 5600$

$\therefore$ Minimize $z = 5920x_1 + 5600x_2$

Grade 1 inspectors = $0 \leq x_1 \leq 9$

Grade 2 inspectors = $0 \leq x_2 \leq 11$

No. of pieces per day: $320x_1 + 240x_2 \geq 2000$

Where $x_1, x_2 \geq 0$

4. An electronic company produces three types of parts for automatic washing machines. It purchases costing of parts from a local foundry and then finishes the part on drilling, shaping and polishing. The selling price of parts A, B and C are Rs. 8, Rs. 10 & Rs. 14 respectively. All parts made can be sold. Costing for parts A, B and C respectively cost Rs. 5, Rs. 8, and Rs. 10. The shop possesses only one of each type of costing machine. Costs per hour to run each of the three machine are Rs. 20 for drilling, Rs. 30 per shaping and Rs. 30 per polishing. The capacities (parts per hour) for each part on each machine are shown in the table.

Machine	Capacity per day		
	part A	part B	part C
Drilling	25	40	25
Shaping	25	20	20
Polishing	40	30	40

The company of the shop to know how many parts of each type it should produce per hour in order to maximize profit for an hour's run. Formulate this problem as L.P. model so as to maximize total profit to the company.

Sol.

	Part A	Part B	Part C
Selling price	8	10	14
costing price	5	8	10
Drilling time (per h.)	25	40	25 → cost Rs. 20
Shapping time (per h.)	20	20	20 → cost Rs. 30
Polishing time (per h.)	30	40	40 → cost Rs. 30

Let,

 x_1 = no. of type A parts produced per h. x_2 = no. of type B parts produced per h. x_3 = no. of type C parts produced per h.

$$\text{Profit Maximize } P = 3x_1 + 4x_2 + 4x_3 - (20 + 30 + 30) \\ = 3x_1 + 4x_2 + 4x_3 - 80$$

Subject to :

$$\text{Drilling} : \frac{x_1}{25} + \frac{x_2}{40} + \frac{x_3}{25} \leq 1$$

$$\text{shapping} : \frac{x_1}{25} + \frac{x_2}{20} + \frac{x_3}{20} \leq 1$$

$$\text{Polishing} : \frac{x_1}{40} + \frac{x_2}{30} + \frac{x_3}{40} \leq 1$$

5 A tape recorder company manufactures models A, B and C. Which have profit contribution per unit of RS. 150, RS. 400 and RS. 600 respectively. The weekly minimum production requirements are 25 units per model A, 130 units for model B and 55 unit per model C. Each type of recorder requires a certain amount of time for the manufacturing of the component parts for assembling and for packing. Specifically a dozen units of 3 hours for assembling and 1 hour of packing. The corresponding figures for a dozen units of model B are 2.5, 4 and 2 and for a dozen units of model C are 6, 9 & 4. During the forthcoming week the company has available 130 h. of manufacturing 140 h. of assembling & 52 h. of packing time, formulate this problem as L.P. model so as to maximize the total profit to the company.

	Model A	Model B	Model C
Profit	150	400	600
Min. production	25	130	55
Manufacturing time	4/dozen	2.5/dozen	6/dozen $\rightarrow 130$
Assembling time	3/dozen	4/dozen	9/dozen $\rightarrow 140$
packing time	1/dozen	2/dozen	4/dozen $\rightarrow 52$

Let,

x_1 = no. of units of model A produce.

x_2 = no. of units of model B produce.

x_3 = no. of units of model C produce.

$$\text{Maximum profit } P = 150x_1 + 400x_2 + 600x_3$$

* Constraints :

$$\text{Manufacturing time} = \frac{4}{12}x_1 + \frac{2.3}{12}x_2 + \frac{6}{12}x_3 \leq 130$$

$$\text{Assembling time} = \frac{3}{12}x_1 + \frac{4}{12}x_2 + \frac{9}{12}x_3 \leq 170$$

$$\text{Packing time} = \frac{1}{12}x_1 + \frac{2}{12}x_2 + \frac{4}{12}x_3 \leq 52$$

$$x_1 \geq 25$$

$$x_2 \geq 130$$

$$x_3 \geq 55$$

Where $x_1, x_2, x_3 \geq 0$.

Date / /

6 A Soft drink plants has two bottling machines A and B. A is designed for 250 ml. bottles and B for 500 ml. bottles. However each can also be used for both types of bottle with some loss of efficiency. The manufacturing data is as follows.

Machines	250 ml. bottle	500 ml. bottle
A	80/min.	30/min.
B	50/min.	60/min.

The machines can be run for 8 hours per day. 5 per week. The profit on an 250 ml. bottle is 5 RS. & on a 500 ml. bottle is 8 RS. Weekly production of the drink can not exceed 2,00,000 bottles and market can absorb 40,000, 250 ml. Bottles and 10,000, 500 ml. bottles per week. The planner wishes to maximize his profit subject to all the production and marketing restrictions. Formulate this problem as an L.P. model to maximize total profit.

sol ⁿ ∴	Machine	250 ml. bottle	500 ml. bottle
	A time	80/min	90/min $\rightarrow 8 \times 5 \times 60$ $= 2400$ min
	B time	50/min	60/min $\rightarrow 8 \times 5 \times 60$ $= 2400$ min
	Profit	5 Rs.	8 Rs.

Let,

x_1 = Weekly production of 250 ml. bottles

x_2 = Weekly production of 500 ml. bottles.

Maximum profit $P = 5x_1 + 8x_2$

* Constraints ∴

Machine A time $= \frac{x_1}{80} + \frac{x_2}{30} \leq 2400$

Machine B time $= \frac{x_1}{50} + \frac{x_2}{60} \leq 2400$

total production $= x_1 + x_2 \leq 2,00,000$